

A continuum model for dislocation dynamics in three dimensions.
constitutive stress rule:

$$\sigma = 2\mu(sym(\nabla \mathbf{u}) + \frac{\nu tr(\nabla \mathbf{u})}{1-2\nu} \mathbf{I} + \sum_{\lambda} \frac{\phi^{\lambda}}{(b^{\lambda})^2} sym(\mathbf{b}^{\lambda} \otimes \nabla \psi^{\lambda}))$$

$$\nabla \cdot \sigma = 0$$

with boundary conditions:

$$\begin{aligned}\mathbf{u}|_{\partial\Omega_d} &= \mathbf{u}^b \\ \boldsymbol{\sigma} \cdot \mathbf{k} &= \mathbf{t}^b\end{aligned}$$

homogeneous example:

$$u_1 = x^2 + y^2$$

$$u_2 = xyz$$

$$u_3 = \frac{1}{2}xy^2 - \frac{1}{2}xz^2 - \frac{2 * A + 6}{A + 1}xz$$

error analysis:

n	$e(u)$	$e(\nabla u)$	$e(a)$
166	0.00072	0.0551	0.0927
965	0.0010	0.0266	0.0423
6385	0.0009	0.0156	0.0213

non-homogeneous example:

$$u_1 = x^2 + y^2$$

$$u_2 = xyz$$

$$\begin{aligned} u_3 = & -\frac{2A+6}{A+1}xz - \frac{1}{2}xz^2 - \frac{C}{A+1}(2z+z^2)x + \frac{C}{A+1}(z^2+4z)y \\ & + \frac{1}{6}x^3 + \frac{C(A+2)}{3(A+1)}x^3 + \frac{1}{2}Cx^2 - \frac{C(A+2)}{3(A+1)}y^3 \end{aligned}$$

error analysis:

n	$e(u)$	$e(\nabla u)$	$e(a)$
166	0.0045	0.0881	0.4896
965	0.0042	0.0448	0.2129
6385	0.0019	0.0230	0.0984

Thank you and good luck!