Counting Arithmetical Structures

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Let G be a finite, simple, connected graph. An arithmetical structure on G is a pair of positive integer vectors d,r such that $(\mathrm{diag}(d)-A)r=0$, where A is the adjacency matrix of G. Arithmetical structures generalize the notion of graph Laplacian and were introduced in the context of arithmetical geometry by Lorenzini in 1989 to model intersections of curves. We investigate the combinatorics of arithmetical structures on certain families of graphs including paths, cycles, and Dynkin graphs, as well as the associated critical groups (the cokernels of the matrices $(\mathrm{diag}(d)-A)$). The arithmetic structures on these graph families display some beautiful combinatorial properties, for example, the arithmetical structures on paths are enumerated by Catalan numbers and are also related to Conway-Coxeter's frieze patterns. In this talk we give an introduction to the topic and an overview of these counting results, including our most recent work on Dynkin diagrams that originated during the 2017 REUF program hosted at ICERM.