Infinite dimensional integration by the multivariate decomposition method

Ian Sloan, University of New South Wales

In this joint work with Frances Kuo, Dirk Nuyens, Leszek Plaskota and Grigorsz Wasilkowski we develop the multivariate decomposition method (MDM) for integrating a real-valued function of an infinite number of variables $x_1, x_2, \ldots$ over a product region. The integrand is assumed to be expressible as a sum of the form

$$f(x) = \sum_{\mathfrak{u}} f_{\mathfrak{u}}(x),$$

where each term $f_{\mathfrak{u}}$ depends only on the variables x_j with $j \in \mathfrak{u}$, and moreover each $f_{\mathfrak{u}}$ is assumed to belong to a normed space $F_{\mathfrak{u}}$, and to satisfy a bound

$$\|f_{\mathfrak{u}}\|_{F_{\mathfrak{u}}} \le B_{\mathfrak{u}},$$

where the $B_{\mathfrak{u}}$ are known constants. The MDM algorithm first determines a finite family of most important sets $\mathfrak{u}$, and for each of these uses an appropriate cubature rule (for example, a Smolyak rule) to approximate the integral of $f_{\mathfrak{u}}$, while approximating the integrals over the remaining terms by zero. The talk will outline the algorithm and the analysis, and suggest that it can be very effective in suitable cases.