On a mild Ito formula for stochastic partial differential equations (SPDEs) and on weak convergence rates for SPDEs with nonlinear diffusion coefficients

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Strong convergence rates for (temporal, spatial, and noise) numerical approximations of semilinear stochastic evolution equations (SEEs) with smooth and regular nonlinearities are well understood in the scientific literature. Weak convergence rates for numerical approximations of such SEEs have been investigated since about 11 years and are far away from being well understood: roughly speaking, no essentially sharp weak convergence rates are known for parabolic SEEs with nonlinear diffusion coefficient functions; see Remark 2.3 in [A. Debussche, Weak approximation of stochastic partial differential equations: the nonlinear case, Math. Comp. 80 (2011), no. 273, 89--117] for details. In this talk we solve the weak convergence problem emerged from Debussche's article and establish esssentially sharp weak convergence rates for different type of temporal and spatial numerical approximations of semilinear SEEs with nonlinear diffusion coefficient functions. Our solution to the weak converge nce problem does not use Malliavin calculus. Rather, a key ingredient in our solution to the weak convergence problem emerged from Debussche's article is a new -- somehow mild -- Ito type formula for solutions and numerical approximations of semilinear SEEs.