Numerical Function Patches for 1D and 2D Real Algebraic Cell Decompositions

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A real algebraic set can be represented as a union of cells, each cell defined implicitly over a patch of local coordinates via continuation from an interior point. Algorithms for computing a cell decomposition of the real points contained in a complex algebraic curve or surface have been implemented in Bertini_real, building on the methods of numerical algebraic geometry as implemented in Bertini. With such a decomposition in hand, one can use continuation to sample each cell to any level of resolution, thereby enabling further processing steps such as triangulation, graphical rendering, and so on. Meanwhile, the Chebfun team has implemented algorithms for numerically computing with functions in Matlab, wherein functions are approximated to machine accuracy using Chebyshev polynomials and related techniques, resulting in a "chebfun". This talk will consider the conversion of the implicit representation of a cell into an explicit "chebfun" representation. Success in building a chebfun representation depends on properly recognizing that the boundaries of cells in a cell decomposition are defined by singularities, and this is especially important in the 2D case. Once a chebfun (or chebfun2) has been constructed, all of Chebfun's capabilities become available, including plotting, root finding, and optimization.