Lowerbounds on semidefinite representations of hyperbolic programs

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Hyperbolic Programming is a generalization of SDP obtained by replacing determinants (whose zero free regions define the PSD cone) by hyperbolic polynomials, whose zero-free regions define hyperbolicity cones. One of the main questions in real algebraic geometry, called the Generalized Lax Conjecture, is whether this generalization is strict -- i.e., whether every hyperbolicity cone can be written as a section of a PSD cone of some (potentially larger) dimension. We study the quantitative version of this question, namely, how large does the dimension of the PSD cone need to be?

We show that there exist (many) hyperbolicity cones in n dimensions of degree d, such that any semidefinite representation of them must have dimension roughly exponential in n^d, even if approximation is allowed. Our cones are random perturbations of the cones induced by the elementary symmetric polynomials. Constructing succinctly describable cones with this property remains an open question. The proof involves robust versions of several classical facts about real rooted polynomials.

Joint with Prasad Raghavendra, Nick Ryder, and Ben Weitz.