Strict Complementarity in Convex Algebraic Geometry (with a focus on Elliptopes)

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Strict complementarity is a refinement of the notion of complementary slackness in optimization. This refined notion leads to various necessary conditions for good-behaviour in convex optimization. We will first review some fundamental results about strict complementarity in the general contexts of convex optimization and convex algebraic geometry. Then, we will focus our attention on linear optimization problems over certain convex sets described by determinantal polynomials; in particular, we will focus on elliptopes.

Linear optimization problems over elliptopes are also called MaxCut SDPs (Semidefinite Programming problems), since they provide very fundamental and useful SDP relaxations for the MaxCut problem. We will see that when a vertex of the feasible region of a MaxCut SDP is optimal, i.e., when the SDP relaxation is tight for the underlying MaxCut problem, strict complementarity fails generically at the boundary of the normal cone of the elliptope at that optimal vertex. In this regard, the MaxCut SDP, and elliptopes, display the nastiest behavior possible for a convex optimization problem.

This talk is based on joint work with Marcel de Carli Silva.