Large deviations in random graphs

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What is the probability that the number of triangles in the random graph $G(n,p)$ exceeds $1+\delta$ times its mean? The works of Chatterjee--Varadhan (dense setting) and Chatterjee--Dembo (sparse setting) reduce this large deviations problem to a natural variational problem on the space of graphons. In this talk, I will discuss some results and challenges concerning the solutions to these variational problems.

Specifically, for the problem of upper tails of triangle counts in $G(n,p)$, in the dense setting (p constant), we have completely determined the replica symmetric phase (i.e., when the random graph conditioned on the upper tail rare event behaves like another Erd\H{o}s--R\'enyi random graph), and in the sparse setting, we have determined the exponential rate of the large deviation probability, at least in the regime $n^{-\alpha} \ll p \ll 1$ where the Chatterjee--Dembo framework applies. Many other cases of the problem remain open.

Joint work with Eyal Lubetzky.