The Phases of Hard Sphere Systems

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One of the simplest statical mechanics models is a system of identical hard sphere particles, placed inside a box whose energy and volume are free to fluctuate in response to an environment characterized by a temperature and pressure. A dimensionless pressure parameter p is formed from a combination of the sphere radius and the temperature and pressure of the environment. Qualitatively different average properties of the sphere packing are controlled by this dimensionless pressure: a "disordered" gas phase at small p and an "ordered" crystalline phase at large p. Since all (nonintersecting) sphere configurations are isoenergetic, the mechanism for crystalline ordering, called "order by disorder", is purely entropic in nature. Sphere packings having the highest possible density correspond to the limit of infinite p.

In addition to providing a physicist's intuition on the existence of phases in the hard sphere system, this tutorial talk will also touch on topics of mathematical interest. In the limit of many spheres (so "magic number" effects are minimized), does statistical mechanics distinguish among the many different densest structures that arise in three dimensions? In higher dimensions, where much less is known, might we expect more than two phases, or perhaps just a single phase?