Statistical mechanics via asymptotics of symmetric functions

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What do lozenge tilings (a.k.a. plane partitions, dimer covers of the hexagonal lattice), alternating sign matrices (or the 6-vertex model) and the dense loop model have in common? For one, their limiting behavior can be studied with the help of some "asymptotic" algebraic combinatorics.

We develop methods to analyze normalized symmetric functions (Schur functions and more general Lie group characters), as the indexing partition converges to a limiting profile. We apply this analysis together with some combinatorial interpretations to study the limiting behavior of the integrable models listed above. In particular, we show that the positions of horizontal lozenges near a vertical flat boundary are distributed like the eigenvalues of GUE matrices, and this holds for a wide class of domains (including such with free boundary). These methods can also be used to establish the existence of limit shapes also for free boundary domains. We discover Gaussian distribution for some observables of the Alternating Sign Matrices, leading again to GUE eigenvalues for the positions of 1s near the border (result of V. Gorin). We also find the asymptotics for the [conjectural] expected value of the mean total current between two adjacent points in the dense loop model.

Based on joint work with Vadim Gorin.