

# Advances in Possible Orders of Circulant Hadamard Matrices and Sequences with Large Merit Factor

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## Abstract

The question of the existence of a circulant Hadamard matrix of order  $n > 4$  is connected to the problem of the existence of a Barker sequence with length greater than 13: a negative answer to the former implies the same answer for the latter. We introduce new methods to improve the search for plausible orders of circulant Hadamard matrices, finding all values less than  $10^{30}$  which pass every known necessary condition. This improves the previous bound by a factor of 2500. We also find a new sequence of polynomials  $\{f_N\} \subset \mathbb{C}[z]$  for which  $\lim_{N \rightarrow \infty} (\|f_N\|_4^4 - \|f_N\|_2^4) / \|f_N\|_2^3 = 4/\pi^2$ , and demonstrate experimentally that for each  $N$ , the merit factor of the coefficient sequence of  $f_N$  exceeds those of the corresponding Px, P1, and Frank sequences. In addition, we discuss a sequence construction related to the Walsh-Hadamard transform that possesses an experimental asymptotic merit factor of 3.