Paths to understanding rowmotion in products of chains

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Birational rowmotion is an action on the space of assignments of rational functions to the elements of a finite partially-ordered set (poset). It is lifted from the well-studied *rowmotion* map on order ideals (equivariantly on antichains) of a poset P, which when iterated on special posets, has unexpectedly nice properties in terms of periodicity, cyclic sieving, and homomesy (statistics whose averages over each orbit are constant). All these actions have been implemented in Sage, and data from experiments has been key to making conjectures and working out proofs.

We give a formula in terms of families of non-intersecting lattice paths for iterated actions of the birational rowmotion map on a product of two chains. This allows us to give a much simpler direct proof of the key fact that the period of this map on a product of chains of lengths r and s is $r+s+2$ (first proved by D. Grinberg and the author), as well as the first proof of the birational analogue of *homomesy along files* for such posets. This work is joint with Gregg Musiker