Numerical Approximations of Fractional Operators using Dunford-Taylor Representations

Andrea Bonito, Texas A&M

We review numerical algorithms based on Dunford-Taylor representations of fractional diffusion problems with a particular emphasis on their analysis and implementations.

In the case of spectral fractional powers of an elliptic operator, a representation of the solution is obtained in term of an improper integral involving solutions to auxiliary, parameter dependent, reaction-diffusion problems. The improper integral is approximated using an exponentially convergent SINC quadrature method. At each quadrature point, a standard finite element method is advocated to approximates the independent auxiliary problems. The method is easily parallelizable and consists of a straightforward modification of standard finite element methods for reaction-diffusion problems.

For the integral fractional laplacian, the Dunford-Taylor integral representation is instrumental to derive novel variational formulations. As in the spectral case, SINC quadrature formulas coupled with finite element discretizations on parameter dependent truncated domains are put in place. This yields a non-conforming method where the action of the stiffness matrix on a vector is approximated (sometimes referred to as a matrix free approach). The efficiency of the method is illustrated in three dimensions.

We then focus on several applications involving either fractional operators and demonstrate how their proposed approximations yield tractable numerical methods even for the approximation of complex systems.