Finite elements for computational electromagnetism from a geometrical point of view

Francesca Rapetti, Université Côte d'Azur

First order edge (resp. face) finite elements on simplices (known also as Whitney finite elements) are widely used finite elements to approximate the electric field (resp. the magnetic induction) for electromagnetic applications. They offer the simplest construction of polynomial discrete differential forms on simplicial complexes. Their associated degrees of freedom (dofs) have a very clear meaning as cochains and, thus, give a recipe for discretizing physical balance laws, e.g., Maxwell's equations.
Higher order extensions of Whitney approximations are receiving increasing interest for their better convergence and accuracy properties but their definition become rather complex. We propose a simple definition for the generators of these finite element spaces, generally known as Nedelec first family of H(curl)- and H(div)-conforming finite element spaces, for polynomial degree 1 and higher than 1, by following a geometrical point of view.

We present how it is possible to define a cardinal basis for these finite element spaces when classical moments of higher order are considered. Here, the basis functions are expressed only in terms of the barycentric coordinates of the simplex. We also discuss on alternative degrees of freedom to the classical moments, namely the physical (natural) weights of the field on particular small-simplices, a set of simplices obtained through affine contractions of a mesh simplex, associated with the principal lattice of the simplex.