An overview of Cold-Boot Attack, related to

RSA and Factorization

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About this talk

Based on the work *"Reconstruction from Random Bits and Error Correction of RSA Secret Parameters"*, jointly done with

Santanu Sarkar

&

Subhamoy Maitra

This extends and supplements the work of *Heninger and Shacham* [Crypto 2009] and that of *Henecka, May and Meurer* [Crypto 2010].

Contents of this talk

- Cold-Boot attack - a brief introduction
- Application 1: Reconstruction of RSA secret parameters
  - Starting from the LSB side [Heninger and Shacham, 2009]
  - Starting from the MSB side [this work]
- Application 2: Error-Correction of RSA secret parameters
  - Starting from the LSB side [Henecka, May and Meurer, 2010]
  - Starting from the MSB side [this work]
- Implications of Cold-Boot attack on RSA - a summary

Cold-Boot Attack

a brief introduction

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> - Halderman et al. [USENIX 2008, Comm. ACM 2009]

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Pieces of the puzzle

- Fact 1: Data remanence in RAM may be prolonged by cooling
- Fact 2: The memory can be dumped/copied through cold-boot
- Fact 3: Memory may retain sensitive cryptographic information

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RSA stores $N, e, p, q, d, d_p, d_q, q^{-1} \bmod p$ in memory (PKCS#1)

Potential information retrieval

- Few random bits of the secret keys $p, q, d, d_p, d_q, q^{-1} \bmod p$
- All bits of secret keys, but with some probability of error

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Question: Does this partial information help the attacker?

Partial Key Exposure attacks on RSA

Rivest and Shamir (Eurocrypt 1985)

N can be factored given 2/3 of the LSBs of a prime.

Coppersmith (Eurocrypt 1996)

N can be factored given 1/2 of the MSBs of a prime.

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What if we know random bits?

Reconstruction

of RSA Secret Parameters

Reconstruction of RSA secret parameters

SITUATION

Cold boot attack provides you with δ **fraction of random bits** *in each secret parameter* p, q, d, d_p, d_q*, where* $0 < \delta < 1$*.*

PROBLEM: Can one correctly reconstruct these parameters?

Reconstruction of RSA secret parameters

Situation

Cold boot attack provides you with δ **fraction of random bits** *in each secret parameter* p, q, d, d_p, d_q*, where* $0 < \delta < 1$*.*

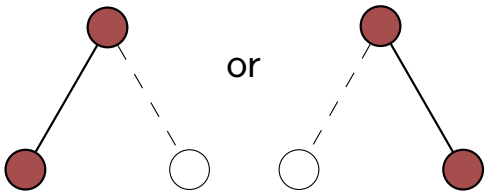
Problem: Can one correctly reconstruct these parameters?

- Heninger and Shacham (Crypto 2009)
  Reconstruction of secret parameters from the LSB side
- Maitra, Sarkar and Sen Gupta (Africacrypt 2010)
  First attempt at reconstruction from the MSB side (known blocks)
- Sarkar, Sen Gupta and Maitra (this talk)
  Reconstruction from the MSB side with known random bits

Heninger and Shacham (Crypto 2009)

- Reconstruction of parameters given δ fraction of random bits.
- Idea: The relation $p[i] \oplus q[i] = (N - p_{i-1}q_{i-1})[i]$ gives a chance for improvised branching and pruning in the search tree

Either $p[i]$ or $q[i]$ is known



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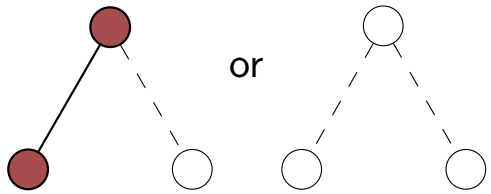

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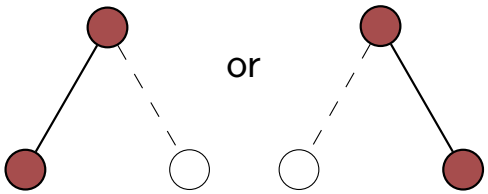

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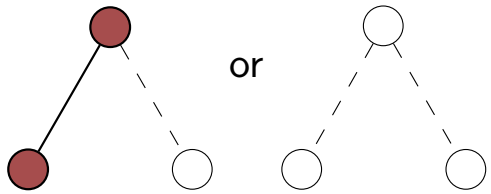
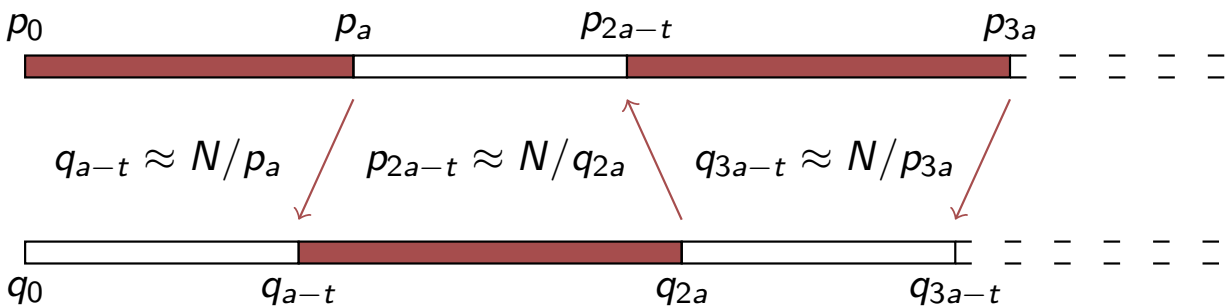


- Result: One can factor N in time poly$(e, \log_2 N)$, given
  - $\delta \geq 0.27$ fraction of random bits of p, q, d, d_p, d_q, or
  - $\delta \geq 0.42$ fraction of random bits of p, q, d, or
  - $\delta \geq 0.57$ fraction of random bits of p, q.

Maitra et al. (Africacrypt 2010)

- Reconstruction of parameters from the MSB side given small blocks of the parameters are known.

- Intuition for primes p, q:



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- Reconstruction of parameters from the MSB side given small blocks of the parameters are known.
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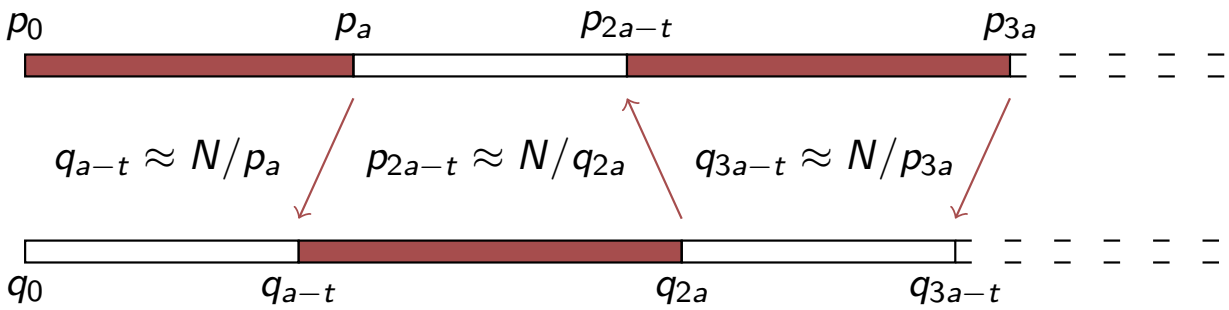

- Result: One can factor N in time $O(\log^2 N)$ with considerable probability of success given $< 70\%$ bits of the primes (together).

Random Bits: Reconstruction of p, q

Context

- We know δ fraction of random bits of both primes p, q
- The goal is to reconstruct prime p from this knowledge

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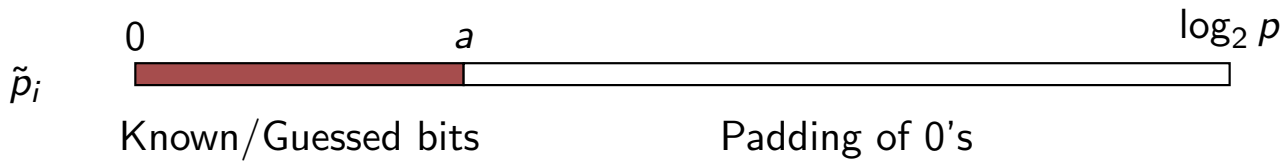
Step 0. Guess Routine

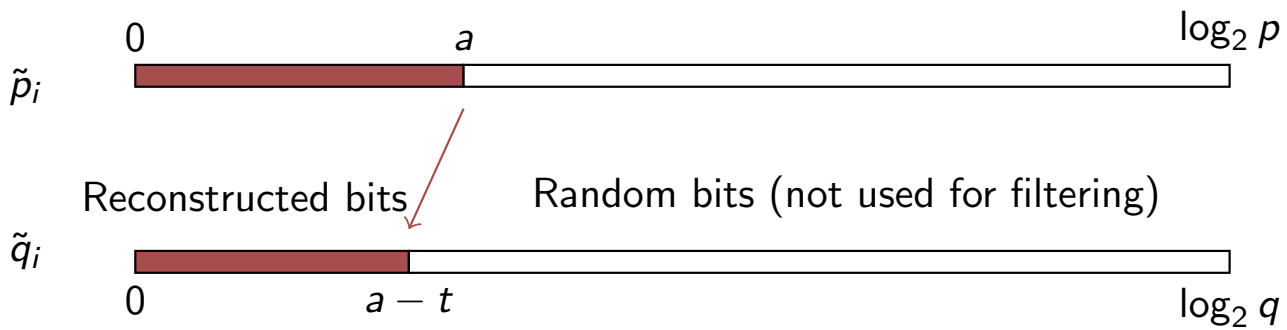
- Generate all $2^{a(1-\delta)}$ options for the first window (a MSBs) in p
- Pad the remaining by 0's, and store in an array A, say.



Random Bits: Reconstruction of p, q

STEP 1. For each option $\tilde{p}_i \in A$,

- Reconstruct first $(a - t)$ MSBs of q using $\tilde{q}_i = \lfloor \frac{N}{\tilde{p}_i} \rfloor$
- Store these options in an array B, say.
- Offset t comes as division is not 'perfect'



Random Bits: Reconstruction of p, q

STEP 2. Filter Routine

- If for some known bit $q[l]$ of q, the corresponding bit in q_i does not match, discard $\tilde{q}_i$ from B, and hence $\tilde{p}_i$ from A.
- If all the known bits of q match with those of $\tilde{q}_i$, retain $\tilde{p}_i$.

$$\text{Filtered } A = \{\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_x\} \text{ where } x = |A| < 2^{a(1-\delta)}$$

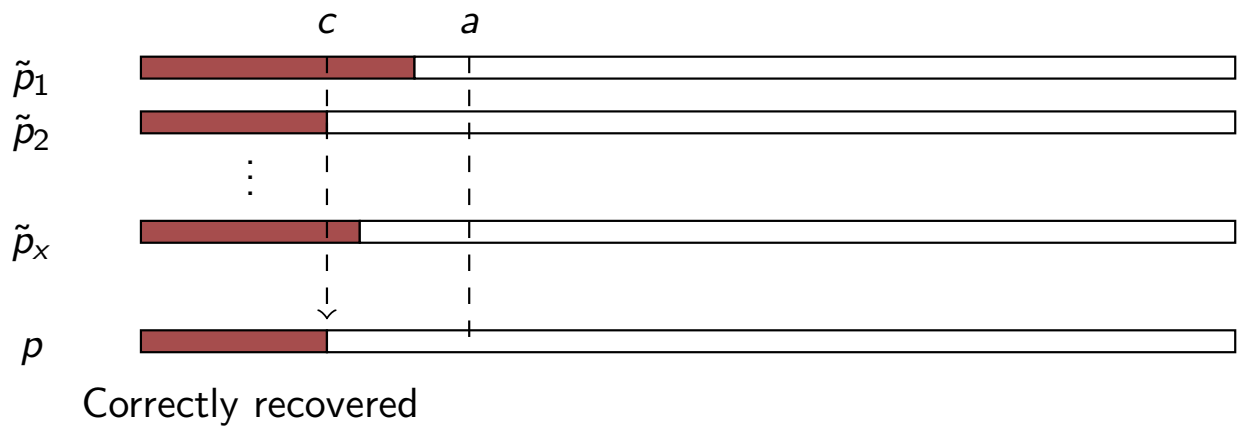
HOPE: Options in A reduce considerably after filtering.

Random Bits: Reconstruction of p, q

Step 3.

- Each option in A has some correctly recovered block of MSBs.
- Find the initial contiguous common portion out of the options

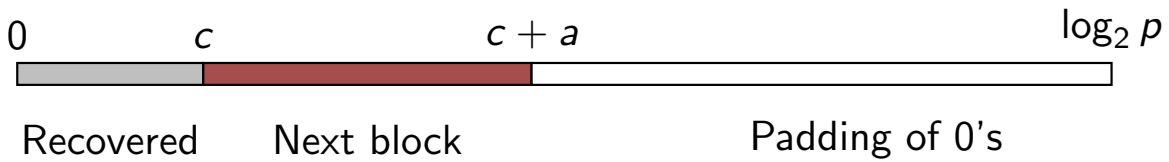
$$\tilde{p}_1[l] = \tilde{p}_2[l] = \cdots = \tilde{p}_x[l] \quad \text{for all } 1 \leq l \leq c, \text{ not for } c < l \leq a$$



Random Bits: Reconstruction of p, q

ITERATE. Slide the Window

- Take next window of a bits of p starting at the $(c+1)$-th MSB
- Repeat Guess and Filter routines using first $(c+a)$ MSBs of p.



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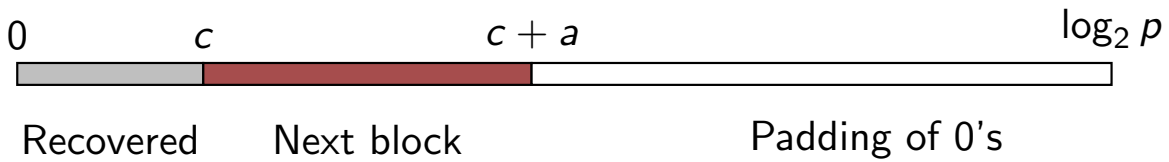

Continue till we get top half of prime p.
Then use Coppersmith's method to factor N efficiently!

Random Bits: Sliding Window Technique

Intuition for the General Algorithm:

1. Fit a window of length a at the top of prime p
2. Find out how many bits we know within this window
3. Guess the remaining unknown bits within the window of a bits
4. Filter through the guesses using the partial information known about the bits of all other secret parameters q, d, d_p, d_q
5. Slide the window forward and continue the same process

Experimental Results

Known	δ	Blocksize a	Offset t	Probability	Time (sec)
p, q	63	30	5	0.3	96
p, q	62	35	5	0.8	379
p, q, d	50	28	6	1.0	831
p, q, d	47	30	6	1.0	10402
p, q, d, d_p, d_q	40	25	6	0.9	2447
p, q, d, d_p, d_q	38	25	6	1.0	3861

We could factor N with considerable success probability, given

- $\delta \geq 0.38$ fraction of random bits of p, q, d, d_p, d_q, or
- $\delta \geq 0.47$ fraction of random bits of p, q, d, or
- $\delta \geq 0.62$ fraction of random bits of p, q.

Comparison with Heninger-Shacham

Heninger-Shacham: LSB side reconstruction with random bits known

Our work: MSB side reconstruction with random bits known

Bits known from	Heninger Shacham	Our result	
		Theory	Experiment
p, q	59%	64%	62%
p, q, d	42%	51%	47%
p, q, d, d_p, d_q	27%	37%	38%

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How do you know the bits for sure?