Mason Experimental Geometry Lab

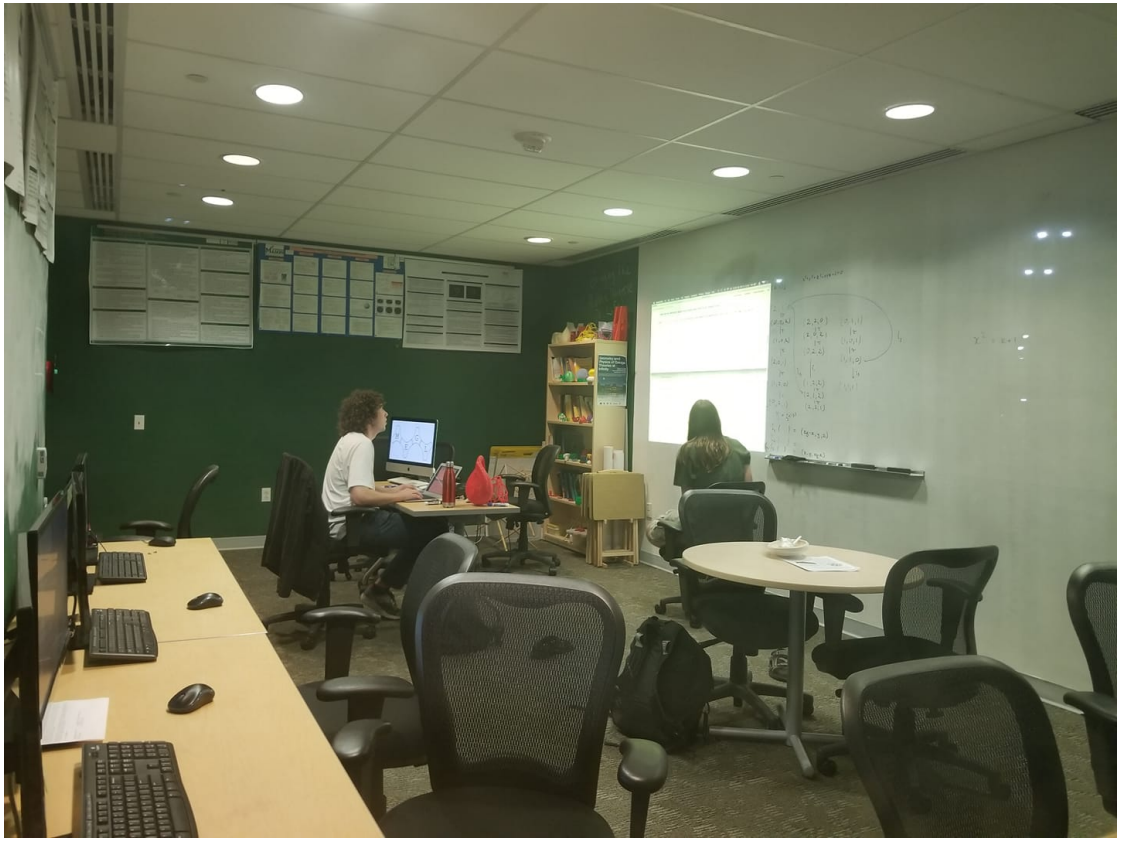
Geometry Labs United 2020

ICERM, July 16, 2020

In the beginning (2014), there was a conference room (obtained by threatening to demolish cubicals):

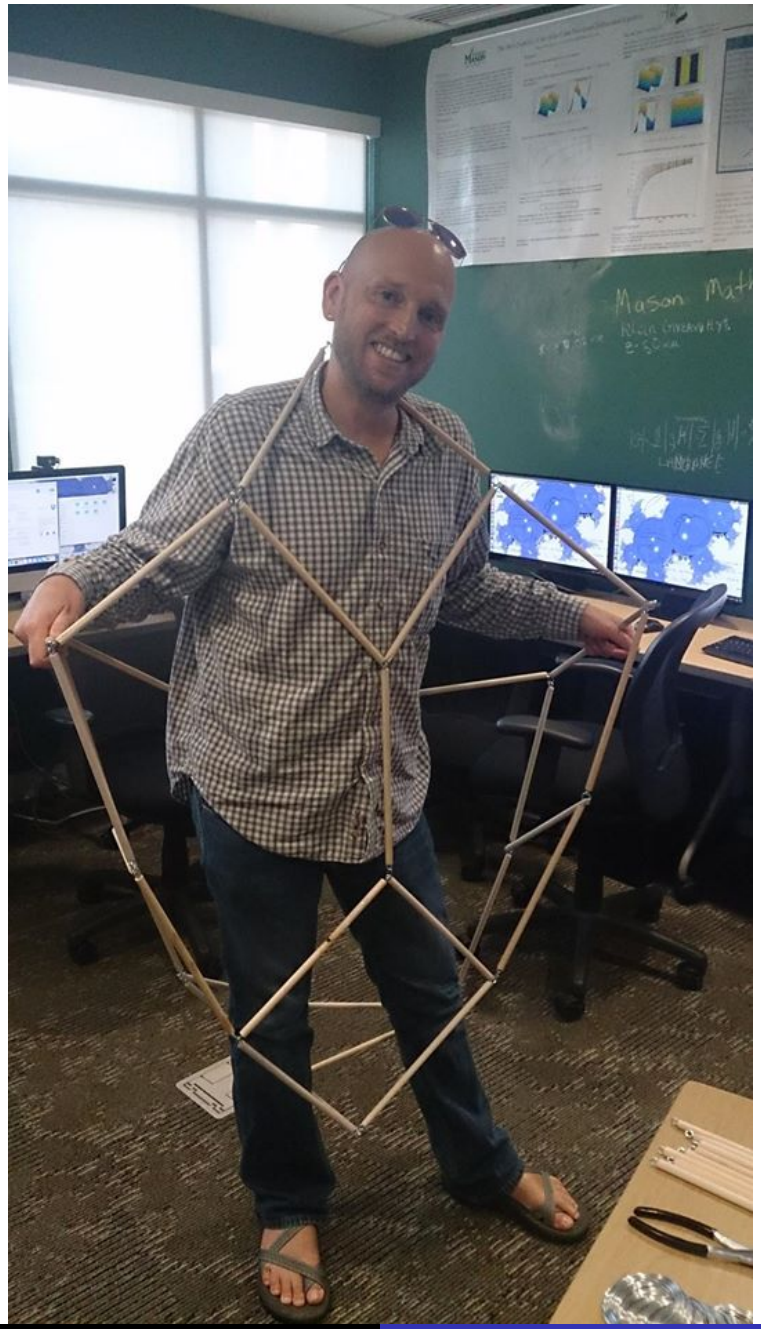
Actual original pic is lost to time...but you get the idea.

With whiteboard and chalkboard paint, a small library, a 3D printer, VR headsets, and a couple fast computers (and a lot of elbow grease) I turned it into this (2014-2019):



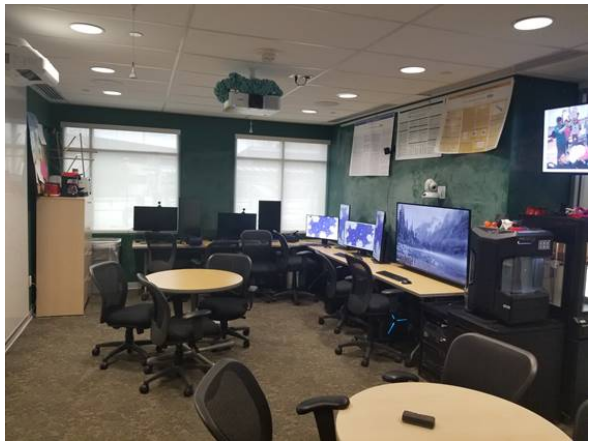
Don't worry, the room was usually more full than the last pic shows:

Sometimes we were silly:



But the goal was to create a diverse community of mathematicians:

With $100,000 from a grant I wrote, the lab was upgraded to this (2019-2020):



We have better VR, two 3D printers, two projectors (one interactive), ceiling cameras (two), ceiling mics, ceiling speakers, central control, better computers (still have a couple “desktop supercomputers”)...

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In short, the state of MEGL is and has been great!

Goodbye & Future

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- I am confident Anton and Harry will do a great job leading MEGL into its future!

What is a Geometry Lab?

> *The* **research community** *at the core of an experimental mathematics lab produces a* **synergistic experience***: the undergraduates' research resonates with their coursework; graduate students' research blends with mentoring; and faculty foster a deeper connection between research and teaching.*

Geometry Labs United: An Invitation, AMS Notices, October 2018

https://www.ams.org/publications/journals/notices/201809/rnoti-p1088.pdf

Geometry Labs United

Presently we have 13 labs in our network (and it is growing with at least 5 more on their way). We hope to expand this network.

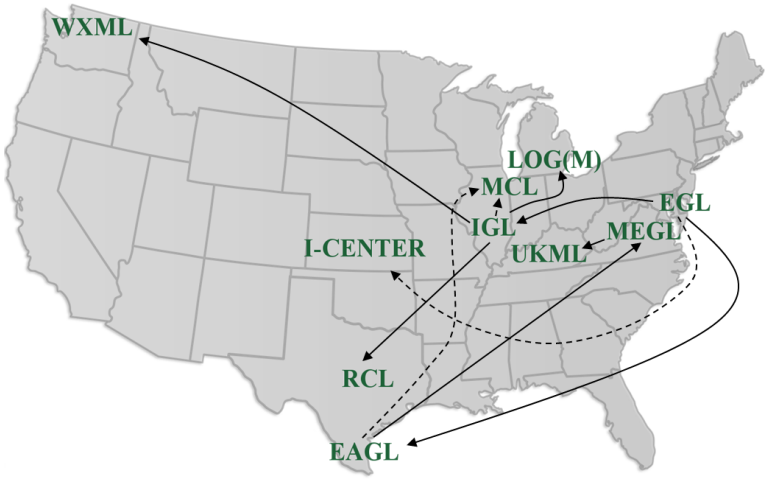

Figure: First ten member labs of the Geometry Labs United (GLU) network.

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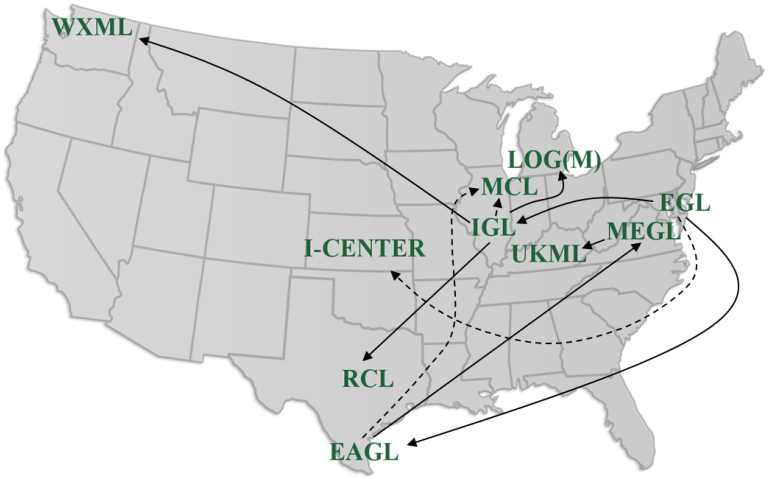


Figure: First ten member labs of the Geometry Labs United (GLU) network.

If you are interested in starting a lab and community, please contact me. I am VERY happy to work with anyone interested in joining GLU.

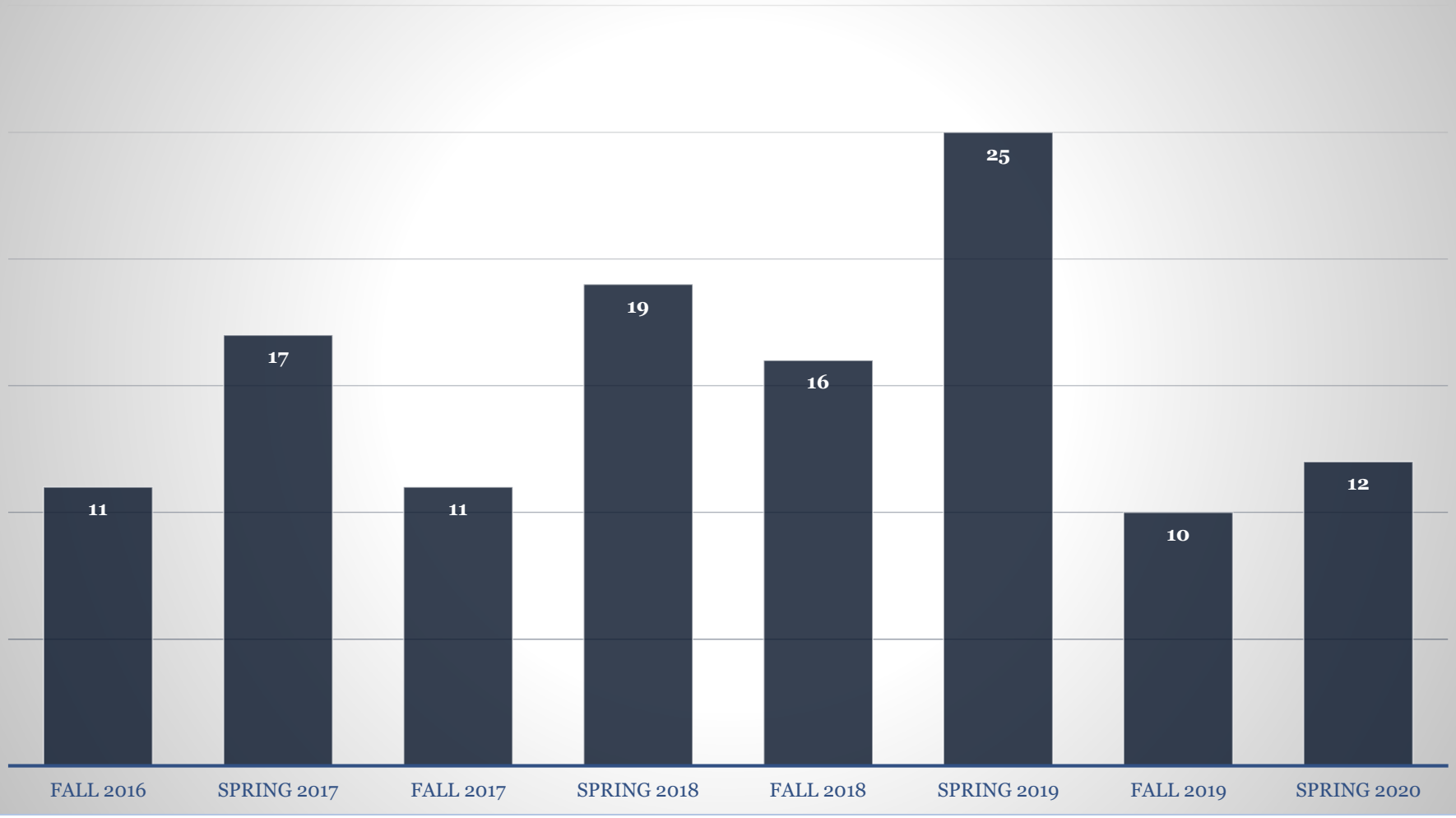
Illinois Geometry Lab

Leadership team:
Madie Farris, Alexi Block Gorman, Philipp Hieronymi, Colleen Robichaux, Brian Shin

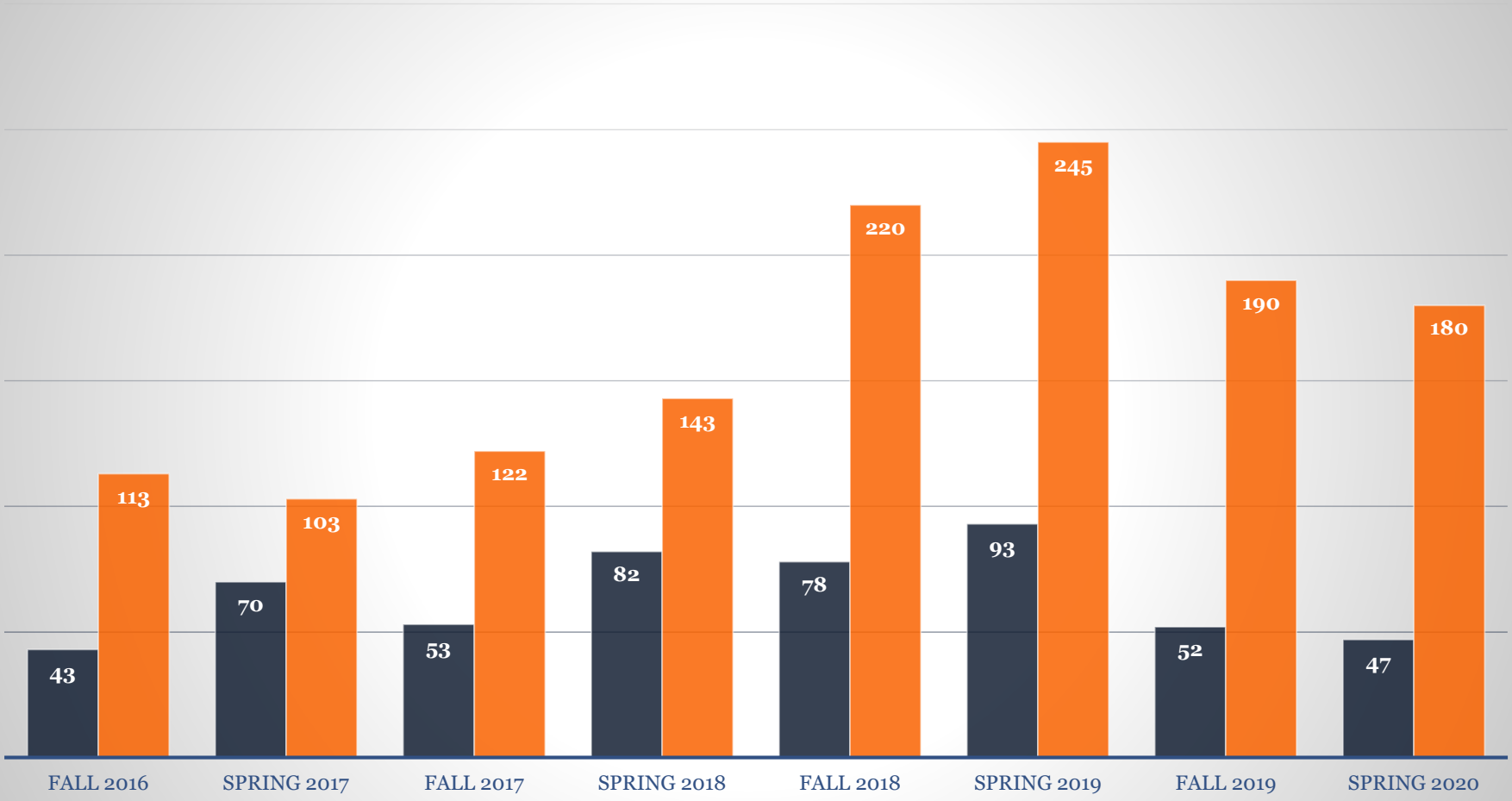
ILLINOIS

https://go.illinois.edu/IGL-Wiki

IGL Projects

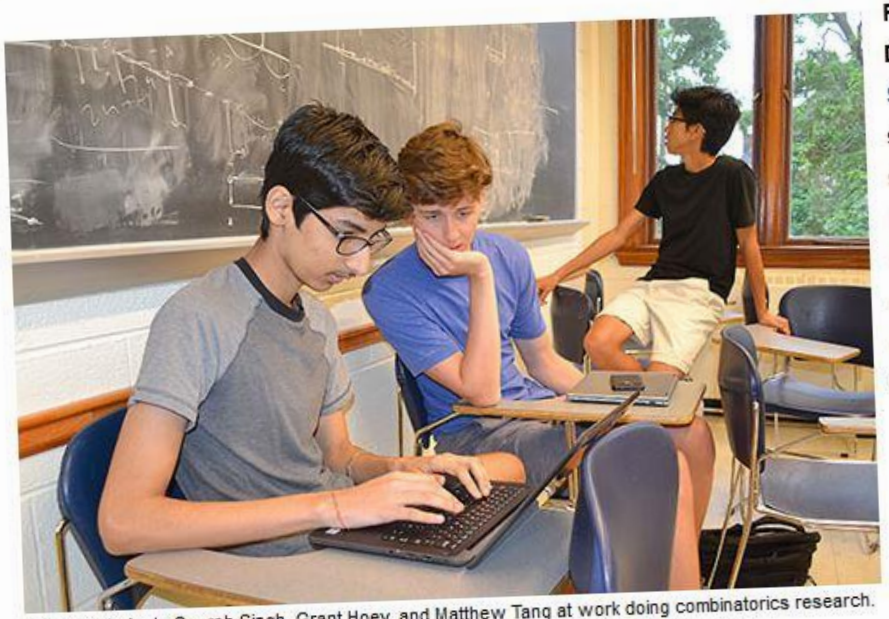


Undergraduate Participants/Applications



The Illinois Geometry Lab Partners with Uni High to Expos Students to Research

June 28, 2019



Uni High students Sparsh Singh, Grant Hoey, and Matthew Tang at work doing combinatorics research.

IGL Director Philipp Hieronymi and Uni High teacher Ioana Boca shed some light

For the second summer in a row, the Illinois Geometry Department of Mathematics partnered with University School (Uni High) to provide research projects for sou students. For four weeks from June 3rd through June schoolers visited Altgeld Hall to conduct math researc Illinois Math graduate students. The idea was to expo students to math not traditionally taught in school anc a taste of what math research is like. In addition, the y experienced another aspect of academia: they prepar then presented their research at a final event on Thu

SIM Campers Experience "Ah-ha! Moments" Over the Beauty, Complexity, Oft-Misunderstood Nature...and Fun...of Mathematics

June 14, 2019

"What is the point of this?!" a middle school student asked several of the 22 Summer Illinois Math (SIM) Camp Epsilon participants during an activity on the first day of the week-long day camp which ran from June 10–14th. Tied into a "human knot," he and several of his fellow campers were trying to get untangled. Here's how they got in this predicament: they stood in a circle facing each other; each raised their right hand and took the hand of someone across from them, then took another's hand with their free hand. The goal? To untangle the knot without letting go of each other's hands, deciding which players should go over, under, backwards, or forwards until they all ended up in a single circle, still holding hands.

The point of the Human Knot activity, according to Jenna Zomback, SIM camp director, in addition to being a fun ice breaker, was to foster teamwork:

SIM camp students participating in a human knot activity.

"We're trying to emphasize group work and communication," she explains, indicating that the activity hadn't necessarily been included to teach the kids any mathematical concepts. **"We just want them to be able to work together to solve problems."** In fact, most of activities throughout the week involved participants solving problems in a group or with a partner.

Winter Math Carnival Adds Up to a Good Time for Local Families

I think this can be a powerful experience for kids who already really like math, but especially kids who are maybe struggling with that and don't think they're good at math to be able to come and have success and fun doing math. I think that can be a really powerful experience. – Elizabeth Field

Young visitors create fractals during the Winter Math Carnival.

February 12, 2020

Math can be fun! This was the idea behind the Winter Math Carnival held at the Alice Campbell Alumni Center on February 2nd from 2:00–5:00 pm. It drew around 150 families and 400–500 people overall—parents, grandparents, and a whole lot of kids having a good time. Sponsored by Illinois Geometry Lab (IGL), a key research/outreach program of Illinois' Mathematics Department, the carnival featured a variety of hands-on, math-related activities and games that encouraged the youngsters to think. Plus, in addition to some goodies, kids had a chance to interact with math students who were eager to share their passion for what they do and how much fun math can be.

Organizing the event were a couple of Illinois Math PhD students. For instance, Simone Claire Sisneros-Thiry got involved because she was the graduate leader for IGL's development of exciting outreach material. Her fellow co-coordinator, Elizabeth Field, has served as a mentor for a couple of undergraduate student research projects. Of course, the Director of IGL, Math Associate Professor Philipp Hieronymi, was also integrally involved.

In fact, one of his duties was advertising the event. The IGL sent tons of flyers to area schools for students to take home to their parents. It was also advertised electronically: through websites like chambanamoms.com, a community website of family-friendly activities, and on Facebook, and also on the math building's

We are extremely grateful for the support, both moral and financial, which we receive from numerous sources. Financial support for the Illinois Geometry Lab comes from the Department of Mathematics, the University of Illinois Office of Public Engagement, the Mathematical Association of America, the American Mathematical Society, the National Science Foundation, and generous gifts from private donors.

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Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

diplomaths

Nick Baeth, Annalisa Crannell
Franklin & Marshall College

New! *Exciting!*

Highlights:
submitted a proposal to the NSF (January 2020).
Have an advisory board and a project evaluator standing at the ready

Huge thank you's to Sean Lawton and to Valparaiso University for encouragement and guidance!

Spring 2020, pilot program:

- 37+ students applied for 12 slots.
- 16 Diplomath Research students: (two teams of four students each, one sophomore and three first-year students; evenly divided in terms of gender.)


Maddie Pfeifer
Kevin Egemba
You
Muhammad Shaamyl Anwar
Tram Nguyen

Factorization in $M_2(\mathbb{N}_0)^\bullet$

X. An, W. Chen, G. Heilbrunn, P. Liu

Advisor: N. Baeth Spring 2020

Motivation

The Fundamental Theorem of Arithmetic tells us that every element of N_0 larger than 1 factors uniquely as a product of primes. However, unique factorization does not hold in many other sets. In this work we consider factorization in the noncommutative semigroup $M_2(\mathbb{N}_0)^\bullet$ of 2×2 material with nonnegative integers entries.

What is $M_2(\mathbb{N}_0)^\bullet$?

- Let $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ denote the set of positive integers and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ the set of nonnegative integers.
- $M_2(\mathbb{N}_0) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{N}_0 \right\}$ is the semigroup of *all* 2×2 matrices with nonnegative integer entries.

Because factorization in noncommutative settings is already interesting enough, we restrict to the cancellative setting, so that if $A, B \neq 0$, $AB \neq 0$. Thus we consider only those matrices with nonzero determinant and study factorization in

$$M_2(\mathbb{N}_0)^\bullet = \{A \in M_2(\mathbb{N}_0) \colon \det(A) \neq 0\}$$

.

Units

Theorem *The two units of $M_2(\mathbb{N}_0)^\bullet$ are* $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ *and* $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Definitions

Let S be a semigroup with binary operation $*$.

- An element ι of S is an *identity* if $s * \iota = s = \iota * s$ for every s in S. Then, an element u of S is a *unit* if there is some v in S so that $u * v = v * u = \iota$. The set of all units is denoted by $S^\times$.
- An element a in S is an *atom* (also called an *irreducible element*) if a is not a unit and if whenever $a = x * y$ for two elements x and y in S, either x is a unit or y is a unit.
- The *length set* of an element s in S is $\mathsf{L}(s) = \{n \colon s = a_1 \cdots a_n\}$. That is, the length set is the set of all possible factorization lengths of the element s in S.
- The *elasticity* of an element s in a semigroup S is $\rho(s) = \frac{\max \mathsf{L}(s)}{\min \mathsf{L}(s)}$, the ratio of the largest factorization length of s to the shortest. Then the elasticity of S is $\rho(S) = \sup_{s \in S}\{\rho(s)\}$. S is fully elastic if for every $q \in Q \cap [1, \rho(S)]$ there is $s \in S$ with $\rho(s) = q$.
- Fix a positive integer k. Then $\mathcal{U}_k = \{n \colon a_1 \cdots a_k = b_1 \cdots b_n\}$. That is, $\mathcal{U}_k$ is the set of all possible factorization lengths of elements that have at least one factorization of length k.
- Elements a and b in a semigroup S are *associates* if $a = ub$ or $a = bu$

Atoms

Theorem *The following matrices and their associates are atoms:*

$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$\begin{bmatrix} p & 0 \\ 0 & 1 \end{bmatrix}$ *where p is prime*

$\begin{bmatrix} 1 & A \\ B & x \end{bmatrix}$ *where $0 < x < A, B$ and $gcd(AB, x) = 1$*

$\begin{bmatrix} 2 & A \\ B & x \end{bmatrix}$ *where $gcd(AB, 2) = 1$; $2 \leq x < A, B$; and $gcd(AB, x) = 1$*

$\begin{bmatrix} 3 & A \\ B & x \end{bmatrix}$ *where $gcd(AB, 3) = 1$; $3 \leq x < A, B$; and $gcd(AB, x) = 1$*

Let $gcd(i, j)$ be the greatest common divisor of i and j.

Theorem *If $\left[\begin{smallmatrix} w & A \\ B & x \end{smallmatrix}\right]$ and its associates are atoms, then the following are true:*

1. $gcd(AB, w) = gcd(AB, x) = 1$
2. *If $w > A$ then $B < x$ or if $w < A$ then $B > x$*
3. *If $w > B$ then $A < x$ or if $w < B$ then $A > x$*

$\mathcal{U}_k(M_2(\mathbb{N}_0)^\bullet)$

Theorem $\mathcal{U}_k(M_2(N_0)^\bullet) = \mathbb{N}_{\geq 2} = \{2, 3, 4, 5, \ldots, n\}$.

Proof: $\mathcal{U}_2(M_2(\mathbb{N}_0)^\bullet) = \mathbb{N}_{\geq 2}$, for all $n \geq 1$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{3^n+1}{2} & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3^n + 1 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}^n \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Also,

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}.$$

Since $n \in \mathcal{U}_k \Leftrightarrow k \in \mathcal{U}_n$ and $n \in \mathcal{U}_k \Rightarrow n + 1 \in \mathcal{U}_{k+1}$, by induction $\mathcal{U}_k(M_2(\mathbb{N}_0)^\bullet) = \{2, 3, 4, 5, \ldots, n\}$ □

Relation to Triangular Matrices

Theorem $T_2(\mathbb{N}_0)^\bullet = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a, b, c \in \mathbb{N}_0 \text{ with } ac \neq 0 \right\}$ *is a semigroup of* $M_2(\mathbb{N}_0)^\bullet$

- If $\left[\begin{smallmatrix} a & b \\ 0 & c \end{smallmatrix}\right] = \mathrm{A}_1 \cdot A_2 \cdot A_3 \ldots . A_n$ in $\mathrm{M}_2(\mathbb{N}_0)^\bullet$, then there are $\mathrm{B}_1 \cdot B_2 \cdot B_3 \ldots . B_n \in \mathrm{T}_2(\mathbb{N}_0)^\bullet$ with $\mathrm{A}_i \sim B_i$ for all i.
- $U_2(\mathrm{T}_2(\mathbb{N}_0)^\bullet) = \{2\} \cup \{P + 1 \ (P \text{ is prime number})\}$
- $M_2(\mathbb{N}_0)^\bullet$ has full infinite elasticity.

For Fall 2020

- five teams among four faculty members
- over 30 students applied for the program

- DRS (Diplomath Research Skills) Seminar: half-credit seminar for all Diplomaths