Negative Snell’s Propagation

Katherine Engelman\textsuperscript{1} and Andrew Kimball\textsuperscript{2}

\textsuperscript{1}Bryn Mawr, \textsuperscript{2}Western Carolina University

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Snell’s Law

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]
Negative Snell’s Law

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta_2) \]
Previous Work
Previous Work

Escaping to Infinity with Triangle Tiling
2 Lines

$\phi$, $\beta$, $l_1$, $l_2$, a, b
$$y_n = b - \beta + n(a - b)$$

$$x_n = \beta + n(b - a)$$
Theorem

A particle can not spiral around the intersection of two lines infinitely.
The number of times the particle hits $l_2$ is $\left\lceil \frac{\beta}{a-b} \right\rceil$.
N-Rays
3-Rays
Square Skewed by $\frac{1}{2}$
Square Skewed by $\frac{1}{2}$

Theorem

Every path in the square brick tiling skewed by $\frac{1}{2}$ is periodic within translation.
Square Skewed by $\frac{1}{2}$

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Square Skewed by $\frac{1}{2}$

Case 1: $e_1 \rightarrow e_4$. 
Square Skewed by $\frac{1}{2}$

Case 2: $e_1 \rightarrow e_5$. 
Case 3: $e_2 \rightarrow e_6$. 
Square Skewed by $\frac{1}{2}$

Case 4: $e_3 \rightarrow e_6$. 
Square Skewed by $\frac{1}{2}$

Case 5: $e_5 \rightarrow e_6$. 
Square Skewed by $\frac{1}{2}$

Case 5: $e_5 \rightarrow e_6$. $\alpha \in \left(0, \arctan\left(\frac{1}{\frac{1}{2} + \lambda}\right)\right)$
Square Skewed by $\frac{1}{2}$

Case 5: $e_5 \rightarrow e_6$. $\alpha \in \left( \arctan \left( \frac{1}{\frac{1}{2} + \lambda}, \pi \right) \right)$
Square Skewed by $\frac{p}{q}$
Square Skewed by $\frac{1}{3}$
Square Skewed by $\frac{1}{3}$
Square Skewed by $\frac{1}{9}$
Parallelogram Tiling
Parallelogram Tiling
Parallelogram Tiling
Parallelogram Tiling
Acknowledgements

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