Equidecomposability and Period Collapse

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August 5, 2014

We begin by classifying the affine unimodular orbits of minimal triangles in the lattice $\frac{1}{d}\mathbb{Z} \times \frac{1}{d}\mathbb{Z}$, which can be understood in terms of an action by the dihedral group on 3 elements. In the case $d = 5$, we observed two triangles with the same Ehrhart quasi-polynomial which were not in the same affine unimodular orbit. We develop an invariant which shows that these two triangles are not equidecomposable, providing a negative answer to a conjecture posed by McAllister and Kantor that Ehrhart equivalence implies equidecomposability. Surprisingly, there does however exist an infinite equidecomposability relation between these two triangles if we delete an edge. In addition, we provide sufficient and necessary conditions for equidecomposability in terms of a family of graphs associated to minimal triangulations of a given polygon. In another direction, we give an explicit formula for the Ehrhart quasi-polynomial in terms of the interior and boundary points up to certain dilates of a polygon. Next, we observe a general linear recurrence relation for the coefficients of the Ehrhart series and give a geometric interpretation for this relation. Under some assumptions, we can do this geometric construction for denominator D triangles with period collapse $k \mid D$, which converts the period collapse problem into studying half-open rational parallelograms whose discrete and continuous areas are the same. We close with some related conjectures and problems.