

Conservation Laws -- Analytical and Numerical Approaches  
Abstracts

**Saturday 10:15 – 12:15**

Charis Tsikkou, Ohio State University  
Helena Nussenzveig Lopes, UNICAMP  
Monica Torres, Purdue University  
Eun Heui Kim, California State University – Long Beach  
Katarina Jegdic, University of Houston, Downtown  
\*5 talks during this session

**Sunday 8:30 – 10:30**

Nathalie Lanson, University of Waterloo  
Chiu-yen Kao, Ohio State University  
Lilia Krivodonova, University of Waterloo  
Ching-Shan Chou, Ohio State University

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Saturday 10:15 – 12:15

**A kinetic equation for an elastic model, via a Hamiltonian lattice with two dimensional displacement field**

Charis Tsikkou, Ohio State University

We work on a harmonic lattice with a nonquadratic on-site potential, and two dimensional displacement field, in the context of non linear elasticity. We use Wigner function, Gaussian decoupling and apply the kinetic theory of L. Boltzmann.

**Weak transport theory and the vortex-wave system**

Helena Nussenzveig Lopes, UNICAMP

The vortex-wave system is a coupling of the two-dimensional vorticity equation with the point-vortex system. It is a model for the motion of a finite number of concentrated vortices moving in a distributed vorticity background. In this talk we discuss existence of a weak solution to this system with an initial background vorticity in  $L^p$ ,  $p > 2$ , up to the time of first collision of point vortices. We also discuss the existence of particle trajectories for this flow.

**On the structure of solutions of nonlinear hyperbolic systems of conservation laws**

Monica Torres, Purdue University

In this talk we show that a Liouville theorem for systems of conservation laws yields the existence of strong traces on hyperplanes of bounded solutions for the one dimensional isentropic Euler equations.

**Global solutions for transonic self-similar 2-dimensional Riemann problems**

Eun Heui Kim, California State University – Long Beach

We discuss the recent development of two-dimensional self-similar transonic Riemann problems. More precisely we discuss analytical results and numerical results on a simplified model system -- the nonlinear wave system.

**Strong regular reflection for two-dimensional isentropic gas dynamics equations**

Katarina Jegdic, University of Houston, Downtown

We consider a Riemann problem for two-dimensional gas dynamics equations that gives rise to strong (or transonic) regular reflection. We write the problem in self-similar coordinates and we obtain a free boundary problem for the reflected shock and a subsonic state. We prove existence of a solution using various fixed point arguments and theory of second order elliptic equations with mixed boundary conditions.

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Sunday 8:30 – 10:30

**Hybrid meshfree methods for hydrodynamics**

Nathalie Lanson, University of Waterloo

Meshfree methods, also referred to as particle methods, are a set of lagrangian methods developed for the approximation of conservation laws in many fields of hydrodynamics and solid dynamics. The main advantage of these methods lays in their ability to handle complex situations involving highly distorted systems, such as crash or impact problems. This is due to the fact that no structural mesh is needed. Despite their potential, meshfree methods present some weaknesses. In particular, the use of numerical viscosity to stabilize the schemes can yield unphysical behaviors. In this talk I will present the hybrid renormalized meshfree scheme. I will show how this scheme is built upon a finite volume treatment of the interactions between particles and present simulations underlining the significant improvement that this new scheme offers.

**BOUNDED DOMAIN PROBLEM FOR THE MODIFIED BUCKLEY-LEVERETT EQUATION**

Chiu-yen Kao, Ohio State University

The focus of the present study is the modified Buckley-Leverett (MBL) equation describing two-phase flow in porous media. The MBL equation differs from the classical Buckley-Leverett (BL) equation by including a balanced diffusive-dispersive combination. The dispersive term is a third order mixed derivatives term, which models the dynamic effects in the pressure difference between the two phases. The classical BL equation gives a monotone water saturation profile for any Riemann problem; on the contrast, when the dispersive parameter is large enough, the MBL equation delivers non-monotone water saturation profile for certain Riemann problems as suggested by the experimental observations. In this talk, we first show that the solution of the finite interval  $[0, L]$  boundary value problem converges to that of the half-line  $[0, +\infty)$  boundary value problem for the MBL equation as  $L \rightarrow +\infty$ . This result provides a justification for the use of the finite interval boundary value problem in numerical studies for the half line problem. Furthermore, we extend the classical central schemes for the hyperbolic conservation laws to solve the MBL equation which is of pseudo-parabolic type. Numerical results confirm the existence of non-monotone water saturation profiles consisting of constant states separated by shocks.

### **Linear Stability Analysis of the Discontinuous Galerkin Method on Non-Uniform Grids**

Lilia Krivodonova, University of Waterloo

Applying a discontinuous Galerkin spatial discretization to a hyperbolic PDE results in a system of ODEs for the unknown solution coefficients. This can be solved with a time integration scheme such as for example a Runge-Kutta method. The largest allowable time step depends on the eigenvalues of the spatial discretization matrix (or the Jacobian in the nonlinear case) and the absolute stability region of the ODE solver. In this talk we present an analysis of the eigenvalues of the DG scheme with the upwind flux applied to the one-dimensional scalar advection equation. We derive a formula for the eigenvalues on an  $N$  element uniform grid in terms of the sub-diagonal Pade  $(p+1, p)$  approximation of  $\exp(-z)$ , where  $p$  is the dimension of the finite element basis. This allows us to draw a number of conclusions about the CFL number and stability of the scheme. For example, we obtain a bound on the largest in magnitude eigenvalue and also its asymptotic growth rate with the order of approximation. Then, we analyze the eigenvalues of the DG on non-uniform grids and demonstrate that a CFL condition less than the one prescribed by the local stability condition can be used.

### **Noise Filtering in Spatial Gradient Sensing and Response during Yeast Cell Polarization**

Ching-Shan Chou, Ohio State University

Cells sense chemical spatial gradients and respond by polarizing internal components. This process is disrupted by gradient noise caused by fluctuations in chemical concentration. In this talk, I will discuss how gradient noise affects

spatial sensing and response. In our study, we discovered that a combination of positive feedback, multiple signaling stages, and time-averaging produced good results. There was an important tradeoff, however, because filtering resulted in slower polarization. Using both modeling and experiments, we showed that yeast cells likely also combine the above three filtering mechanisms to achieve impressive spatial-noise tolerance, but with the consequence of a slow response time.