Fabrizio Catanese:Â **Projective K(\pi,1) spaces and applications to moduli problems**

An interesting Â theme of research Â is the study of projective varieties Z which are K(H,1)'s , i.e. classifying
spaces BH for some discrete group H.Â For Â such varieties Z, a bold conjecture is that Â also their Galois conjugates Z^s are classifyingÂ spaces BH' for some discrete group H' .Â The well known examples are: curves and abelian varieties, the latter being exactly theÂ projective K(\pi,1) spaces where the group \pi is abelian.Â Interesting examples are the Bagnera-De Franchis varieties and Generalized Hyperelliptic varieties,Â quotients A/G of an Abelian variety A by a finite group G.Â Hypersurfaces in BdF varieties are special cases of the notion of Inoue type varieties,Â whose moduli spaces have been investigated in our joint work with I. Bauer and D. FrapportiÂ (especially for the classification of algebraic surfaces with low invariants).Â Inoue type varieties are defined as Â the quotient X = W/G of an ample divisor W in a Â projective varieties Z which is a Â K(H,1),Â by the free action of a finite group G.
In order to obtain general results on the moduli spaces of ITV, implying thatÂ if X' is homotopically equivalent to an ITV X, Â also X' Â is an Inoue type variety,Â we need to extend the definition to multiple Inoue type varieties.Â This is done via a theorem, recently proven in joint work Â with Yongnam Lee,Â giving an explicit characterization of Â deformations to embeddings as smooth hypersurfaces.

Time permitting, I shall also discuss a Â remarkable series of algebraic surfaces, counterexamples to Fujita's question on VHSÂ (in joint work with Michael Dettweiler).

THM. If one has a Â Kaehler family fibred over a curve B, then the direct image V of the relative dualizing sheafÂ is the direct sum of an ample vector bundle A and of a unitary flat vector bundle W. V does not need to be semiample,Â equivalently, the bundle W can have infinite monodromy.

The examples are given by surfaces S which Â are abelian coverings Â with group Â (\ZZ/n)^2 of the Del Pezzo surface Z of degree 5,Â branched on a union of lines which forms a bianticanonical divisor.

The Albanese map a of S Â is a semistable fibration onto a curve B of genus b, Â with Â fibres of genus g = (n-1)/2, and where g=2b.Â a has only 3 singular fibres, the union of two smooth curves of genus b.

The simplest case is for n=5, where Â S is a ball quotient. Â Do all Â these surfaces have negative curvature, are they projective K(\pi,1)'s?