HÃ©lÃ¨ne Esnault:Â **Some Fundamental Groups in Arithmetic Geometry**

Content:Â 1) Deligneâ€™s conjectures:Â â„“-adic theory
Â Â Â Â Â Â Â Â 2) Â Deligneâ€™s conjectures: crystalline theory
Â Â Â Â Â Â Â Â 3) Maltsev-Grothendieck theorem; Gieseker conjecture; de Jong conjecture
Â Â Â Â Â Â Â Â 4) Relative 0-cycles

1) We review Deligneâ€™s Weil IIÂ â„“-adic conjecture 1.2.10, its analogous finiteness Â statement in Hodge theory (Deligne â€™87), on what (Lefschetz type) theorems the solution the Weill II conjecture relies, for the existence of weights (how to go from Lafforgueâ€™s theorem in dimension 1 to higher dimension), ofÂ â„“'-companions (solved by Drinfeld) Â and for finiteness (solved by Deligne).Â

2) We review Deligneâ€™s Weil II p-adic conjecture 1.2.10, mention Abeâ€™s recent analogue of Lafforgue theorem, and explain what Lefschetz theorems are missing to go further, even to show the existence of weights.Â

3) We present a â€˜conservativityâ€™ program, the origins of which are rooted in Maltsev-Grothendieck theorem: in general one may ask what corresponds, over a perfect field of characteristic p>0, to complex local systems. We discuss the category of crystals we treat, mention Gieseker conjecture and its positive solution Â for crystals on the infinitesimal site (Esnault-Mehta), de Jong conjecture and its partial positive solution for isocrystals coming from crystals in the crystalline site (Esnault-Shiho).Â

4) We pose the problem of a motivic version of the GrothendieckÂ â„“-adic base change theorem and give a partial answer for relative 0-cycles (Kerz-Esnault-Wittenberg).