

Andrei Okounkov: Enumerative geometry and representation theory

My goal in these lectures is to explain how geometric representation theory of certain interesting infinite-dimensional algebras interacts with Donaldson-Thomas theory, which is an enumerative theory of sheaves on a nonsingular threefold X . Enumerative information may be collected in many different formats, and our focus will be on computations in equivariant K-theory, with equivariant cohomology providing a blueprint. In the first lecture, my plan is to motivate the interest in K-theoretic enumerative geometry by discussing a conjectural relation between K-theoretic DT counts to counts of curves in certain associated Calabi-Yau 3-folds, as well as certain remarkable dualities that K-theoretic counts are expected to satisfy. In the second lecture, I will review the structure of DT theory and relate its building blocks to representation theory of certain infinite-dimensional quantum groups related to double loop algebras. The third lecture will be about the geometry behind the construction of these quantum groups. The central notion here will be the notion of a stable envelope in equivariant cohomology, equivariant K-theory, and, eventually, equivariant elliptic cohomology. My lectures will be based on joint work with Mina Aganagic, Roman Bezrukavnikov, Davesh Maulik, Nikita Nekrasov, and Andrei Smirnov.