Nick Sheridan: Counting curves using the Fukaya category

In 1991, string theorists Candelas, de la Ossa, Green and Parkes made a startling prediction for the number of curves in each degree on a generic quintic threefold, in terms of periods of a holomorphic volume form on a `mirror manifold'. Givental and Lian, Liu and Yau gave a mathematical proof of this version of mirror symmetry for the quintic threefold (and many more examples) in 1996. In the meantime (1994), Kontsevich had introduced his `homological mirror symmetry' conjecture and stated that it would `unveil the mystery of mirror symmetry'. I will explain how to prove that the number of curves on the quintic threefold matches up with the periods of the mirror via homological mirror symmetry. I will also attempt to explain in what sense this is `less mysterious' than the previous proof.