Kirsten Wickelgren: Splitting varieties for triple Massey products in Galois cohomology

The Brauer-Severi variety $a x^{\wedge} 2+b y^{\wedge} 2=z^{\wedge} 2$ has a rational point if and only if the cup product of cohomology classes associated to $a$ and $b$ vanish. The cup product is the order-2 Massey product. Higher Massey products give further structure to Galois cohomology, and more generally, they measure information carried in a differential graded algebra which can be lost on passing to the associated cohomology ring. We show that the variety $X$ defined by b x^2 = (y_1^2-ay_2^2 + c y_3^2-ac y_4^2) $)^{\wedge} 2-c(2$ y_1 y_3-2 a y_2 y_4)^2 with $x$ invertible is a splitting variety for the triple Massey product, and give examples of rings over which X has no points following Jochen GÃartner. We then show that this variety satisfies the Hasse principle. It follows that all triple Massey products over global fields vanish when they are defined. More generally, one can show this vanishing over any field of characteristic different from 2; JÃ ${ }_{i n} \operatorname{Min} \tilde{A}_{j} \ddot{A}$ and Nguyá»...n Duy TÃøn, and independently Suresh Venapally, found an explicit rational point on $X(a, b, c)$. $\operatorname{Min} \tilde{A}_{j} \not ̈ A$ and $T A \tilde{A} \not \subset n$ have other nice results in this direction. The method could produce splitting varieties for higher order Massey products. This is joint work with Michael Hopkins.

