A Feynman Approach to Dynamic Rate Markov Processes

William Massey, Princeton University Department of Operations Research & Financial Engineering

Physics has a well-known history of successfully applying mathematical techniques to the description of nature. What makes its history exceptional has been its ability to broaden this language by inspiring the creation of new mathematics. The development and use of calculus along with differential equations by Isaac Newton to invent 17th century physics is a prominent example of this positive feedback loop.

We can apply more recent mathematical physics techniques to understand the random evolution of time-inhomogeneous Markov process or more simply dynamic rate Markov processes. We first use the 19th century physics concept of a Hamiltonian for the special case of time-homogeneous or constant rate Markov processes. Here the Kolmogorov forward and backward differential equations that govern the dynamics of their transition probabilities are mathematically equivalent to the Heisenberg and Schrödinger pictures of early 20th century quantum mechanics.

Richard Feynman (Princeton PhD in 1942) created a new formulation of quantum mechanics during the middle in the 20th century. This framework incorporates both of these quantum pictures but also includes relativistic effects. He did so by reintroducing the 18th century physics concept of a Lagrangian. After that, he introduced an operator calculus to formulate the time-ordered exponential. By applying these techniques to dynamic rate Markov processes, we can transcend the analysis of their transition probabilities and reveal a more fundamental sample path structure.