Elastic Graphs

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Graphical spines can be used to give a convenient combinatorial description of rational maps (or more generally branched self-covers). From this graphical description, there is a family of energies E^p,
depending on a parameter $1 \le p \le \infty$, whose asymptotic behaviour determines properties of the Julia set:
\begin{itemize}
\item For $p = \infty$: Whether the map is expanding (and thus has a reasonable Julia set);
\item For $p = 2$: Whether the map is a rational map;
\item For $p = 1$: Whether the Julia set is “carpet-like”; and
\item More generally, information about the Ahlfors regular conformal dimension of the limit set.
\end{itemize}

Portions of this work are joint with Jeremy Kahn and Kevin Pilgrim.