Workshop on Arithmetic Geometry, Number Theory, and Computation
Lightning Talks
June 2, 2020
Session I

The structure of the group of rational points of an abelian variety over a finite field
Caleb Springer, Penn State

The group of rational points of an abelian variety over a finite field can be viewed as a module over the endomorphism ring of the abelian variety. From this viewpoint, Lenstra showed that, in the case of elliptic curves, this module structure has a nice description in terms of the Frobenius endomorphism. In this talk, I will present a generalization of Lenstra's result to abelian varieties of arbitrary dimension whose endomorphism rings satisfy certain nice properties.

Rigidity in Elliptic Curve Local-Global Principles
Jacob Mayle, University of Illinois at Chicago

Katz established a local-global principle for elliptic curves over a number field that have nontrivial ℓ-torsion locally everywhere. Sutherland gave an analogous local-global principle for elliptic curves that admit a rational ℓ-isogeny locally everywhere. We study these ``locally everywhere'' conditions, showing that a failure of either must be rather significant.

Superelliptic curves with large Galois images
Pip Goodman, University of Bristol

In Serre's famous 1972 paper he proves that an elliptic curve without complex multiplication has maximal image at all but finitely many primes. Since then much work has been done on extending this result, mainly to higher dimensional abelian varieties with trivial endomorphism ring, but also to cases where the endomorphism algebra permits one to view the Galois representation as having low dimension, e.g., those of GL_2 type.
In this talk I will present a result which allows one to construct superelliptic jacobians with maximal images outside a small explicit set of primes. I will then outline the various new tools required, which come from a delightful blend of group theory and the theory of complex multiplication.

Gluing curves along their 2-torsion
Jeroen Hanselman, Universität Ulm

In this talk we will discuss two methods to construct a genus 3 curve whose Jacobian is isogenous to the product of the Jacobian of a given genus 1 curve and a given genus 2 curve.

Rational Points of Fermat Quartics

Oana Adascalitei, Boston University In the 1990s J.-P. Serre challenged the mathematical community to find all the rational points of the Fermat quartic given by x^4 + y^4 = 17. This problem had resisted the attacks of the existing methods of Chabauty-Coleman and Manin-Demjanenko and was solved in 2001 by V. Flynn and J. Wetherell. Their method involves a covering collection technique which may cast light on other similar Fermat quartics, for example x^4 + y^4 = 97.

Explicit arithmetic of superelliptic curves and jacobians

Vishal Arul, MIT

I will quickly describe the main results of my thesis on division by 1-zeta and on torsion points on superelliptic curves. I will state a few words about the methods and ideas for future directions.

Separating periods of quartic surfaces

Emre Sertöz, Leibniz University Hannover

Kontsevich--Zagier periods form a natural number system that extends the algebraic numbers by adding constants coming from geometry and physics. Because there are countably many periods, one would expect it to be possible to compute effectively in this number system. This would require an effective height function and the ability to separate periods of bounded height, neither of which are currently possible.

In this talk, we introduce an effective height function for periods of quartic surfaces defined over algebraic numbers. We also determine the minimal distance between periods of bounded height on a single surface. We use these results to prove heuristic computations of Picard groups that rely on approximations of periods. Moreover, we give explicit Liouville type numbers that can not be the ratio of two periods of a quartic surface. This is ongoing work with Pierre Lairez (Inria, France)."