Arithmetic of twists of supersingular abelian varieties

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We consider a supersingular abelian variety A defined over a finite field K, together with its K-twists. The Weil numbers of A determine A up to isogeny; the K-twists of A may have different Weil numbers.

We say that A is maximal (resp. minimal) over K if all its normalised Weil numbers over K are -1 (resp. +1), since then A has a maximal (resp. minimal) number of K-points.
We ask when A becomes maximal over a finite extension of K. If it does, we say A has parity 1, and if it does not, then A has parity -1.
We study how twisting affects the parity and ask whether A has a K-twist with parity 1. For low-dimensional supersingular abelian varieties, we give a complete answer to this question, under conditions on their automorphism group.

This is joint work with Rachel Pries.