Problem 1 (Kiran Kedlaya). Let $X_3 \subseteq \mathbb{P}^{n+1}_{\mathbb{F}_q}$ be a smooth cubic hypersurface. For which $n \in \mathbb{Z}_{\geq 2}$ and $q$ prime power is it guaranteed that $X$ contains a line (over $\mathbb{F}_q$)?

This is known for $n = 2$ and all $q$ by the theory of cubic surfaces: the answer is negative for all $q$. For $n \geq 5$, the answer is always affirmative by Debarre–Laface–Roulleau. For $n = 3, 4$, there are open cases.

If the answer is not always affirmative, we could ask more generally for the probability for fixed $n, q$, or say as $q \to \infty$. (Wanlin Li)

The probability ought to be be 1 or nearly so for $q$ sufficiently large. We already know this for $n = 5$ and above, so only $n = 3$ and $n = 4$ are at issue. The Fano variety of lines must have rational points for $q$ large enough, except possibly for very special cubics for which the variety decomposes into Galois conjugates—can that happen at all? (Noam Elkies)

Problem 2 (Isabel Vogt). Let $X_3 \subseteq \mathbb{P}^{n+1}_{\mathbb{F}_q}$ be a smooth cubic hypersurface and let $S \subseteq X(\mathbb{F}_q)$ be a finite subset of rational points (with $\# S \leq q + 1$). Does there exist nonconstant $f: \mathbb{P}^1_{\mathbb{F}_q} \to X$ such that $S \subseteq f(\mathbb{P}^1(\mathbb{F}_q))$?

It is known if $q$ is large enough for fixed $n$.

For $\# S = 1$ and cubic surfaces, it is unknown (for $q \leq 8$).

Problem 3 (Vladimir Dokchitser). Let $E$ be an elliptic curve over $\mathbb{Q}$. Let $K \supseteq \mathbb{Q}$ be a cyclic cubic extension and $L \supseteq K$ a cyclic extension with $[L:K] = 21$, and suppose $L \supseteq \mathbb{Q}$ is Galois and nonabelian. Let $\chi: \text{Gal}(L|K) \hookrightarrow \mathbb{C}^\times$ and consider the function $L(E_K, \chi, s)$ obtained as the $L$-function of $E$ base-changed to $K$ twisted by $\chi$.

Conjectures imply that $3 \mid \text{ord}_{s=1} L(E_K, \chi, s)$. Using modularity of $E$, can you prove this?

Problem 4 (Chris Rasmussen). There are several places where existing Sage code for solving $S$-unit equations could be optimized—please help!

A target application would be to list all genus 2 curves over $\mathbb{Q}$ with good reduction away from 3 (Drew Sutherland); it might be more tractable to do those with at least one rational Weierstrass point (Noam Elkies).

Problem 5 (Yuri Zarhin, 1989). Let $X$ be a smooth projective variety over a number field $K$. Is there a positive density set of primes $p$ over $K$ such that $X \mod p$ is ordinary, i.e., the Newton polygon of the action of $\text{Frob}_p$ acting on $H^i_{\text{et}}(X, \mathbb{Q}_\ell)$ is equal to the Hodge polygon for all $i$.

This is known for projective spaces, elliptic curves, abelian surfaces, Mumford abelian fourfolds, CM abelian varieties (in all dimensions), K3 surfaces, and as well as products of these.

Is there a single example of an absolutely simple, typical abelian threefold for which this is known? (Kiran Kedlaya)
Problem 6 (Ben Smith). Consider the directed graph $G_1$ with vertices given by supersingular elliptic curves $\mathbb{F}_p$ up to isomorphism and directed edges given by 2-isogenies. The graph $G_1$ is connected.

Now consider the graph $G_2$ with vertices principally polarized superspecial abelian surfaces over $\mathbb{F}_p$ (i.e., isomorphic to a product of supersingular elliptic curves in an unpolarized way) with edges $(2,2)$-isogenies. Is this graph connected? This has been checked for $p \leq 1000$.

Are the graphs Ramanujan? More generally, what are the spectral properties of $G_2$? (Noam Elkies)

For $G_1$, the graph is known to be connected by strong approximation for quaternions. Does this work for $G_2$?

For $E$ ordinary over $\mathbb{F}_p$, consider the graph whose vertices are given by principal polarizations on $E \times E$ and directed edges given by $(2,2)$-self isogenies. This graph is connected.

Problem 7 (Davide Lombardo). Let $A$ be a abelian surface over a number field $K$ that is typical $(\text{End}(A) = \mathbb{Z})$. By Serre, for each prime $\ell$, the $\ell$-adic Galois representation $\rho_{\ell} : G_K \to \text{GSp}_4(\mathbb{Z}_\ell)$ has open image. Can you bound the index effectively in terms the height of $A$, $[K : \mathbb{Q}]$, $d_K$?

When $\ell$ is large, one can see that the the index is 1 (and there is a formula for this).

Does it help if $K = \mathbb{Q}$? (Nils Bruin)

Problem 8 (Bjorn Poonen). Is there a reasonably practical algorithm to compute the geometric endomorphism ring of an abelian variety over finitely generated fields $K$, such as global function fields? Such an algorithm exists for finite fields and number fields.

In general, because of the relation between Néron–Severi groups and endomorphism rings of abelian varieties, a paper by Poonen, Testa, and van Luijk implies the existence of a horribly slow algorithm for computing endomorphism rings.

Problem 9 (Edgar Costa). Let $A$ be an abelian variety over a number field. Is there a reasonable algorithm that can access the zeta function of $A$ modulo $p$ for arbitrary $p$ and the height of $A$ (or the conductor of the $L$-function) and returns as output the geometric endomorphism algebra?

Work of Chris Hall may be helpful (Jeff Achter).