A discontinuous Galerkin scheme for front propagation with obstacles

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We are interested in front propagation problems in the presence of obstacles. We extend our previous work to propose a simple and direct discontinuous Galerkin (DG) method adapted to such front propagation problems. We follow the formulation of Bokanowski et al leading to a level set formulation driven by $\min(u_t + H(x,\nabla u), u-g(x))=0$, where $g(x)$ is an obstacle function. The DG scheme is motivated by the variational formulation when the Hamiltonian H is a linear function of ∇u, corresponding to linear convection problems in presence of obstacles. The scheme is then generalized to nonlinear equations, written in an explicit form. Stability analysis are performed for the linear case with Euler forward, a Heun scheme and a Runge-Kutta third order time discretization using the technique proposed in Zhang and Shu. Several numerical examples are provided to demonstrate the robustness of the method. Finally, a narrow band approach is considered in order to reduce the computational cost.