A Scalar Conservation Law with discontinuous flux for supply chains with finite buffers

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A heuristic aggregate continuum model for production flows and supply chains with finite buffers is proposed and analyzed. The model extends earlier partial differential equations that represent deterministic coarse grained models of stochastic production systems based on mass con- servation. The finite size buffers lead to a discontinuous clearing function describing the throughput as a function of the work in progress (WIP). Following previous work on stationary distribution of WIP along the production line, the clearing function becomes dependent on the production stage and decays linearly as a function of the distance from the end of the production line. A transient experiment representing the breakdown of the last machine in the production line and its subsequent repair is analyzed analytically and numerically. Shock waves and rarefaction waves generated by blocking and reopening of the production line are determined. Comparisons with discrete event simulations of the same experiment are made.
Elements for a first principle derivation of the PDE models are discussed.