Analysis of PDE models for neuronal networks

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Many integro-differential equations are used to describe neuronal networks or neural assemblies. Among them, the Wilson-Cowan equations are the most well known and describe spiking rates in different locations. Another classical model is the integrate-and-fire equation that describes neurons through their voltage using a particular type of Fokker-Planck equations. It has also been proposed to describe directly the spike time distribution, which seems to encode more directly the neuronal information. This leads to a structured population equation that describes at time t the probability to find a neuron with time s elapsed since its last discharge.

We will compare these models and perform some mathematical analysis. A striking observation is that solutions to the I\&F can blow-up in finite time, a form of strong desynchronization. We can also show that for small or large connectivity, the 'elapsed time model' leads to desynchronization. For intermediate regimes, sustained oscillations can occur which are compatible with observations. A common tool is the use of the relative entropy method.

This talk is based on works with K. Pakdaman and D. Salort, M. Caceres and J. A. Carrillo.