

The jam equation: a functional-differential equation arising from a traffic flow model

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I will explain how macroscopic extensions of Aw-Rascle traffic models can be derived from Vlasov-type kinetic models. If the inherent non-locality of driver behaviour is taken seriously, these new models contain non-localities and delay terms. Preliminary numerical studies suggest that for reasonable initial conditions traveling (braking or acceleration) waves form, and this motivates a travelling wave ansatz. In the simplest case, for a braking wave only, the emerging momentum equation can be transformed into the unusual functional-differential equation

$$\left[\begin{array}{l} (z(s) + \alpha)^2 z'(s) = \beta (z(s+z(s)) - z(s)), \\ \end{array} \right]$$

where α and β are positive parameters. Clearly, every constant is a solution. The interesting question is whether or not this equation admits non-constant decreasing solutions, and I will present a number of numerical and analytical results to this end.