Kac's Program in Kinetic Theory

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The talk is devoted to present some recent results obtained in collaboration with M. Hauray, C. Mouhot and B. Wennberg concerning the propagation of chaos and the mean-field limit for systems of indistinguable particles undergoing collision processes, as formulated by M.~Kac in 1956 for a simplified model and extended by H. P. McKean in 1967 to the
Boltzmann equation.

More specifically, with C. Mouhot, we prove quantitative and uniform in time estimates measuring the distance between the many-particle system and the limit system. These estimates imply in particular the propagation of chaos for marginals in weak measure distances but are more general: they hold for non-chaotic initial data and control the complete many-particle distribution. We also prove the propagation of {\it entropic chaos}, as defined by Carlen, Carvalho, Loss, Le Roux and Villani, answering a question of Kac about the microscopic derivation of the H-theorem.

We finally prove estimates of relaxation to equilibrium (in Wasserstein distance and relative entropy) \emph{independent of the number of particles}. Our results cover the two main Boltzmann physical collision processes with unbounded collision rates: hard spheres and \emph{true} Maxwell molecules interactions.

Starting from an inspirative paper of A. Gr\"unbaum from 1971 we develop a new method which reduces the question of propagation of chaos to the one of proving a purely functional estimate on some generator operators ({\em consistency estimates}) together with fine differentiability estimates on the flow of the limit non-linear equation ({\em stability estimates}).

These results provide the first answer to the question raised by Kac of relating the long-time behavior of a collisional particle system with the one of its mean-field limit, however using dissipativity at the level of the mean-field limit instead of using it at the level of the many-particle Markov process.