A Dynamic Bi-Orthogonal Stochastic Method for Solving Time-Dependent Stochastic PDEs

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Many physical and engineering problems involving uncertainty enjoy certain low-dimensional structures, e.g., in the sense of Karhunen-Loeve expansions (KLEs). In this talk, we propose a Dynamic Bi-Orthogonal Stochastic Method (DyBO) to solve SPDEs by closely following the KL expansions of the original stochastic solutions. The KL expansions are known to be the most compact representations of stochastic processes in an L2 sense. Our methods explore such sparsity and offer computational saving over some existing methods such as generalized Polynomial Chaos methods (gPC). We demonstrate the efficiency of our method through various numerical examples ranging from spatially one-dimensional examples, such as stochastic Burgers' equations and stochastic transport equations to spatially two-dimensional examples, such as Rayleigh-Bernard convection with stochastic forcing. Parallelization is also discussed, aiming toward future industrial-scale applications. In addition to numerical examples, theoretical aspects of DyBO are also carefully analyzed, such as preservation of bi-orthogonality, error propagation, and computational complexity.