Statistical Models of Spatial Processes Based on Local-Interaction Energy Functionals

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This presentation will focus on statistical models of spatially distributed processes based on random fields generated by energy functionals that involve local interactions (Spartan spatial random fields -SSRFs). I will first talk about practical applications, e.g. environmental monitoring and estimation of natural resources, which can benefit from novel mathematical approaches for the processing of spatial and spatiotemporal data. Then, I will summarize the classical geostatistical approach for modeling spatial data while also providing motivation for the introduction of local-interaction based models. Subsequently, I will focus on a specific family of Gaussian SSRFs that incorporate the square gradient and curvature in their energy functional. I will discuss mathematical SSRF properties, as well as model inference, spatial interpolation (prediction) and simulation in this framework. SSRF covariance models are characterized by the sparse structure of the precision (inverse covariance) matrix, at least for lattice data. The sparseness derives from the locality of the operators in the respective energy functional and leads to explicit spectral forms. In certain cases, the correlations in real space can be derived analytically by direct integration of the spectral representation given by the Hankel transform of the spectral density. The availability of explicit expressions for both the covariance and the precision matrix can overcome the curse of dimensionality in the numerical procedures of parameter inference, interpolation and conditional simulation of spatial data. Time permitting I will present a method for estimating geometric anisotropy based on an identity derived by Swerling, and an extension of the interaction-based concept to spatial data with non-Gaussian probability distributions which involves discretized random fields with Ising-type "spin" interactions. Finally, I will discuss topics for further research such as robust expressions for scattered data, and the development of non-homogeneous random fields and space-time models based on local interaction functionals.

Some references:

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