Wiener Chaos Expansion for Random Perturbations in SPDEs

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We discuss random perturbations in linear elliptic and parabolic SPDEs and the stochastic Navier-Stokes equation, based on Malliavin divergence operators and the Wick product. This approach interprets products as stochastic convolutions and is equivalent to the Ito formulation of SDEs, and furthermore has nice properties that make the random perturbation unbiased, even with the nonlinearity in the stochastic Navier-Stokes equation. Additionally, the Wiener chaos expansion and the availability of an adjoint operator, namely the Malliavin derivative, facilitate the analysis of solutions and numerical error estimates. We will see that the Wiener chaos expansion provides an efficient algorithm for numerical simulation, due to the lower triangular structure of the associated PDE system, and then compare numerical simulations of our stochastic model of the Navier-Stokes equations with the standard stochastic Navier-Stokes model.