

Ferromagnets and the mean-field classical Heisenberg model

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There are two main statistical mechanical models of ferromagnetism: the simpler and better-understood Ising model, and the more realistic and more challenging classical Heisenberg model, where the spins are in the 2-sphere instead of in $\{-1,+1\}$. In dimensions one and two, the classical Heisenberg model with nearest-neighbor interactions has no phase transition, but in three dimensions it has been intractable.

To shed some light on the qualitative behavior of the 3D Heisenberg model, we use the versatile tools of mean-field theory and Stein's method in recent work with Elizabeth Meckes, studying the Heisenberg model on a complete graph with the number of vertices going to infinity. Our results include detailed descriptions of the magnetization, the empirical spin distribution, the free energy, and a second-order phase transition. I will also discuss work in progress simulating related models.