Slow Mixing for the Hard-Core Model on Z^2

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The hard-core model has attracted much attention across several disciplines, representing lattice gases in statistical physics and independent sets in discrete mathematics and computer science. On finite graphs, we are given a parameter λ, and an independent set I arises with probability proportional to $\lambda^{|I|}$. We are interested in determining the mixing time of local Markov chains that add or remove a small number of vertices in each step. On finite regions of Z^2 it is conjectured that there is a phase transition at some critical point λ_c that is approximately 3.79. It is known that local chains are rapidly mixing when $\lambda < 2.3882$. We give complementary results showing that local chains will mix slowly when $\lambda > 5.3646$ on regions with periodic (toroidal) boundary conditions and when $\lambda > 7.1031$ with non-periodic (free) boundary conditions. The proofs use a combinatorial characterization of configurations based on the presence or absence of fault lines and an enumeration of a new class of self-avoiding walks called taxi walks.