Computational surprises: mathematical programming and normal surface theory

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Normal surfaces allow us to express topological problems using the framework of high---dimensional polytopes and integer lattices, a natural setting for integer and linear programming. We are now beginning to explore the practical potential of such mathematical programming techniques in real software, and the results so far have been both delightful and surprising. We discuss applications ranging through unknot recognition, vertex surface enumeration, Hakenness testing, and crosscap numbers for knots. The underlying techniques are rigorous, and are customised for normal surfaces by combining hand---crafted heuristics, topological techniques and mathematical programming knowhow. Using these tools we are able to consistently and programmatically certify properties such as knottedness and incompressibility in bulk, with computations taking just minutes for triangulations with up to 50 tetrahedra.

This is based on joint work with Melih Ozlen, and also includes work with Alex Coward and Stephan Tillmann.