

Localization of the eigenfunctions and associated free boundary problems

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The phenomenon of wave localization permeates acoustics, quantum physics, energy engineering. It was used in the construction of noise abatement walls, LEDs, optical devices. Localization of quantum states of electrons by a disordered potential has become one of the prominent subjects in quantum physics, as well as harmonic analysis and probability. Yet, no methods predict specific spatial location of the localized waves.

In this talk I will present recent results revealing a universal mechanism of spatial localization of eigenfunctions of an elliptic operator in a bounded domain. Via a new notion of "landscape" we connect localization to a certain multi-phase free boundary problem, indicate specific location, shapes, and frequencies of localized eigenmodes, and establish regularity (uniform rectifiability) and other geometric properties of the emerging subregions. We shall also discuss peculiarities of the decay of eigenfunctions, dependence on the boundary, the operator, the potential, as well as some "inverse" design problems.

This is joint work with D. Arnold, G. David, M. Filoche, and D. Jerison.