Generalized multilevel methods for accelerating hierarchical stochastic collocation approximations of PDEs with random input data

Clayton Webster, Oak Ridge National Laboratory

Multilevel methods seek to decrease computational complexity by balancing spatial and stochastic discretization errors. As a form of variance reduction, multilevel Monte Carlo (MLMC) techniques have been developed, but can be extended to accelerate stochastic collocation (SC) approximations. In the first part of this talk we introduce a generalized multilevel SC (MLSC) technique, and present a detailed convergence and complexity analysis, demonstrating the advantages of the MLSC compared to standard single level approximations, and highlight conditions under which the MLSC approach is preferable to the MLMC. Moreover, we also present an approach to adaptively accelerate a sequence of hierarchical interpolants required by the MLSC approach. Taking advantage of the hierarchical structure, we build new iterates and improved preconditioners, at each level, by using the interpolant from the previous level. We also provide rigorous convergence analysis of the fully discrete problem and demonstrate the increased computational efficiency, as well as bounds on the total number of iterations used by the underlying deterministic solver.