Optimal algorithms for doubly weighted approximation of smooth functions

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We consider a ρ-weighted L^q approximation in the space of functions $f : \mathbb{R}_+ \to \mathbb{R}$ with $\|f^{(r)}\,\psi\|_{L^p} < \infty$. Let $\alpha = r - 1/p + 1/q$ and $\omega = \rho/\psi$. Assuming ψ and ω are non-increasing and the quasi-norm $\|\omega\|_{L^{1/\alpha}} < \infty$ we construct algorithms using function/derivatives evaluations at n points with the worst case errors proportional to $\|\omega\|_{L^{1/\alpha}}\, n^{-r+(1/p-1/q)_+}$. In addition we show that this bound is sharp; in particular, if $\|\omega\|_{L^{1/\alpha}} = \infty$ then the rate $n^{-r+(1/p-1/q)_+}$ cannot be achieved by any algorithm using n points. Our results generalize known results for bounded domains such as $[0,1]$ and $\rho = \psi \equiv 1$. We also provide a numerical illustration.