Compressive Sampling of Sparse Polynomial Chaos Expansions: Convergence Analysis and Sampling Strategies

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Realistic analysis and design of multi-disciplinary engineering systems requires not only a fine understanding and modeling of the underlying physics and their interactions but also recognition of intrinsic *uncertainties* and their influences on the quantities of interest. Uncertainty Quantification (UQ) is an emerging discipline that attempts to address the latter issue: It aims at a meaningful characterization of uncertainties from the available measurements, as well as efficient propagation of these uncertainties through the governing equations for a quantitative validation of model predictions.

This talk provides a brief introduction to uncertainty propagation using spectral methods, specifically Polynomial Chaos (PC) expansions, along with numerical challenges associated with these methods. Following that, recent developments on sparse PC approximation via compressive sampling, specifically l_1-minimization, will be introduced as a means to tackle these difficulties. The rest of the talk focuses on the convergence analysis and random sampling of PC expansions for successful solution recovery via l_1-minimization. In particular, it will be shown that bounding a probabilistic parameter, referred to as *coherence*, yields a bound on the number of samples necessary to identify coefficients in a sparse PC expansion. Utilizing properties of orthogonal polynomials, bounds on the coherence parameter will be provided for polynomials of Hermite and Legendre type under their respective natural sampling distribution. These bounds reveal that sampling the solution of interest from the probability distribution of the random inputs may be sub-optimal, in the sense that they may lead to large coherence values. In both polynomial bases importance sampling distributions will be identified, which yield bounds on coherence with weaker dependence on the order of the approximation. For more general orthonormal bases, the so-called coherence-optimal sampling will be introduced, which directly uses the basis functions under consideration to achieve a statistical optimality among all sampling schemes with identical support. Numerical examples will be provided to illustrate the effect of different sampling strategies.

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