Exponential decay of reconstruction error from binary measurements of sparse signals

Simon Foucart, University of Georgia

Binary measurements arise naturally in a variety of statistical and engineering applications.
They may be inherent to the problem---e.g., in determining the relationship between genetics and the presence or absence of a disease---or they may be a result of extreme quantization.
A recent influx of literature has suggested that using prior signal information can greatly improve the ability to reconstruct a signal from binary measurements. This is exemplified by one-bit compressed sensing, which takes the compressed sensing model but assumes that only the sign of each measurement is retained. It has recently been shown that the number of one-bit measurements required for signal estimation mirrors that of unquantized compressed sensing. Indeed, s-sparse signals in $\R^n$ can be estimated (up to normalization) from $\Omega(s \log (n/s))$ one-bit measurements. Nevertheless, controlling the precise accuracy of the error estimate remains an open challenge. In this paper, we focus on optimizing the decay of the error as a function of the oversampling factor $\lambda := m/(s \log(n/s))$, where m is the number of measurements. It is known that the error in reconstructing sparse signals from standard one-bit measurements is bounded below by $\Omega(\lambda^{-1})$. Without adjusting the measurement procedure, reducing this polynomial error decay rate is impossible.
However, we show that an adaptive choice of the thresholds used for quantization may lower the error rate to $e^{-\Omega(\lambda)}$. This improves upon guarantees for other methods of adaptive thresholding as proposed in Sigma-Delta quantization. We develop a general recursive strategy to achieve this exponential decay and two specific polynomial-time algorithms which fall into this framework, one based on convex programming and one on hard thresholding. This work is inspired by the one-bit compressed sensing model, in which the engineer controls the measurement procedure. Nevertheless, the principle is extendable to signal reconstruction problems in a variety of binary statistical models as well as statistical estimation problems like logistic regression.