Measure estimation from moments: from line spectra to tensors

Gongguo Tang, University of Wisconsin

Modern data analysis relies on identifying complex, nonlinear models from noisy and incomplete data. Many such tasks can be formulated and analyzed as linear moment problems in infinite-dimensional measure spaces, and the "nice" instances can be solved numerically using semidefinite programming. I will illustrate these ideas using two examples: line spectral estimation from incomplete moment information and 3rd-order symmetric tensor decomposition, which is the estimation of a measure supported on a high-dimensional sphere from its 3rd-order moments. In both examples, the true measure is recovered by minimizing the total mass of the measure, an infinite-dimensional analog of L1 minimization. The recovery procedure is guaranteed to work as long as the support of the true measure is scattered. For line spectral estimation, the infinite-dimensional total mass minimization has an exact SDP formulation, while for tensor problems there is a hierarchy of SDPs with progressively tighter approximations. However, under the same condition that the infinite-dimensional optimization works, we show that the lowest order relaxation in the hierarchy is guaranteed to produce the correct tensor decomposition. We hope this work could shed light on how to perform tensor completion in a theoretically optimal way using a single, small-size semidefinite program.