(Quasi) Monte Carlo and Partial Differential Equations - what can be done?

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The problem with numerical integration is that the function might be big in those places, where we don't sample. If we sample in nicely distributed points and the function still insists on being big in between, it has to have a large derivative/total variation - the Koksma-Hlawka inequality is a neat description of this insight. However, even for wildly oscillating functions with a large total variation not all hope is lost: one could still take N points randomly and get an approximation with an expected error of $N^{-0.5}$. The purpose of this talk is to describe a simple but curious special case: on somewhat weird domains in the plane, harmonic functions - even though they may oscillate arbitrarily wildly - can be integrated slightly faster than random at speed $N^{-0.536}$. This poses many questions: given a partial differential equation, are there integration rules for its solution whose speed of convergence is faster than $N^{-0.5}$ unconditionally (i.e. without introducing a control on total variation)?