Asymptotic Results on the Discrete Riesz Minimal Energy and Polarization when the Power of the Potential Equals the Dimension of the Set

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The talk features the recent joint work with Doug Hardin and Edward Saff (Vanderbilt University, USA) and with Nattapong Bosuwan (Mahidol University, Thailand).

We will discuss our resent constructions of asymptotically s-energy minimizing sequences of N-point configurations as well as the asymptotic behavior of the s-polarization problem (as N gets large) on d-dimensional bodies and manifolds when $s=d$.

It turns out that asymptotic d-energy minimizing property of a sequence of N-point configurations on a d-dimensional set is conditional upon the uniformness of the distribution and well-separation of the sequence.

In particular, we show that the sequence of configurations on a compact set in R^d (of positive Lebesgue measure and boundary of zero measure), which is obtained by intersecting the set with any full-rank lattice scaled by a factor going to zero is asymptotically d-energy minimizing. When the boundary of the d-dimensional conductor has a positive Lebesgue measure, an algorithm for constructing an asymptotically d-energy minimizing sequence is described.

Furthermore, we show that on a compact subset of a d-dimensional smooth manifold, any asymptotically best-packing sequence of N-point configurations is asymptotically d-energy minimizing. We remark that the leading term of the minimal d-energy was earlier found by D.P. Hardin and E.B. Saff on compact subsets of R^d and of d-dimensional smooth manifolds.

The N-point d-polarization problem, which is dual to energy minimization, requires finding an N-point configuration on a conductor with the largest minimum over the conductor of the Riesz d-potential. We find the leading term of the maximal N-point d-polarization on a compact subset of a d-dimensional smooth manifold as well as the weak-star limit distribution of optimal configurations.