Convolution operators, measures of polynomial growth, and finite point configurations.

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We study $L^p(\mu) \to L^q(\nu)$ mapping properties of the convolution operator $T_\lambda f(x) = \lambda * (f\mu)(x)$, where λ is a tempered distribution, and μ and ν are compactly supported measures satisfying the polynomial growth bounds $\mu(B(x,r)) \leq Cr^{s_\mu}$ and $\nu(B(x,r)) \leq Cr^{s_\nu}$. As a significant application of this work, we prove variants of the classical L^p-improving (Littman; Strichartz) and maximal (Stein) inequalities in a setting where the Plancherel formula is not available. Another particularly motivating application is to the study of geometric configurations in subsets of Euclidean space of a given Hausdorff dimension.