**Isogenies, polarisations and real multiplication**

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Isogenies are an essential tool in Elliptic Curves Cryptography, where they are used in a wide variety of area: fast point counting, complex multiplication methods, speeding up the arithmetic... Modular polynomials allow to explore the isogeny graph which has a well known volcano structure (for ordinary elliptic curves over a finite field).

When (A,L) is a principally polarised abelian variety (ppav), and K a finite kernel of A, the abelian variety A/K may not be principally polarised when A has dimension greater than one. In this talk we will explain the relationship between isogenies of principally polarised abelian varieties, polarisations and real multiplication.

We will focus on the case of abelian surfaces over finite field, where there are two types of isogenies between ppav. The first one are isogenies where the kernel is maximally isotropic for the Weil pairing; they correspond to the descent of a power of the principal polarization and are anologous to the isogenies between elliptic curves. The second type are isogenies with a cyclic kernel; they only occur with a maximal real multiplication and correspond to a polarisation which is not a power of a principal polarisation.

We will describe the two types of isogenies, how to compute modular polynomials for both type, and outline the structure of the isogeny graph for principally polarised abelian surfaces; we will see that we need to use both type of isogenies to construct this isogeny graph.